

## Review Problems for Final Exam

Note: This review covers topics from Sections 5.2 & 5.3, as well as a variety of topics presented throughout the semester. This review does not cover every concept that could be tested on a final exam. Please also take a look at the previous Week-in-Reviews for more practice problems.

1. If Louis deposits \$50 at the end of each month into a savings account earning interest at a rate of 7.25% per year compounded monthly, how much will he have at the end of 30 years (assuming that he makes no withdrawals during that period)? How much interest will he earn?
2. You wish to retire with \$1,000,000 in a retirement account, which you will make equal monthly deposits to during the 45 years that you work. If the account will pay interest at a rate of 5% per year compounded monthly, how much should you deposit each month in order to have your million? What is the effective annual yield of this account?
3. You are looking to buy a new car and have only \$1000 for a down payment. The car you wish to buy has a cash price of \$22,500. If the best financing option you find charges interest at a rate of 3.5% per year compounded monthly, how big would your monthly payments be in order to pay off the car in 60 months, assuming you use the money you already have for a down payment? In 48 months? In 36 months?

4. Ten years ago Quincy made a down payment on a house of 20% of the purchase price and secured a bank loan of \$72,500 to finance the remaining amount. The mortgage was for a term of 30 years, with an interest rate of 7.25% per year compounded monthly on the unpaid balance to be amortized through equal monthly payments.

(a) What is the outstanding principal on Quincy's house now?

(b) How much equity does Quincy have in the house now?

(c) How much total interest will Quincy pay over the life of the loan?

(d) At this time, interest rates have dropped to 5% per year compounded monthly on a 15-year mortgage and Quincy is thinking about refinancing. If he refinances, what will his new monthly payments be?

(e) How much money, if any, will Quincy save by refinancing (assuming no additional refinancing costs)?

5. Sally buys a new \$1000 television by paying \$100 and financing the remaining \$900. The terms of her finance agreement state that the unpaid balance will be charged interest at a rate of 15% per year, compounded monthly, and the money is to be repaid over a 2 year period through equal installments made at the end of each month.

(a) What will Sally's monthly payments be?

(b) How much of the first payment goes towards paying down the loan?

(c) Fill in the first 6 lines of the amortization schedule.

End of Period	Pmts Remaining	Payment	TO Interest	TO Principal	Outstanding Principal	EQUITY
0						
1						
2						
3						
4						
5						

(d) How much equity will Sally have in her computer after 1 year?

6. A company making radios finds that the total cost of producing 100 radios is \$9,000 and that the total cost of producing 150 radios is \$13,000. Each radio sells for \$110. Let  $x$  be the number of radios made and sold. Find the

(a) cost function.

(b) revenue function.

(c) profit function.

(d) break-even point and explain its meaning.

7. Use the Method of Corners to solve the following:

OBJ: Max  $P = 2x + 5y$

SUBJ TO:  $x + y \leq 10$

$$3x + y \geq 12$$

$$-2x + 3y \geq 3$$

$$x \geq 0, y \geq 0$$

8. Given  $U = \{0, 1, 2, \dots, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$ ,  $B = \{2, 3, 4, 5, 6\}$ , and  $C = \{4, 8, 10\}$ , find the following sets.

(a)  $A \cup B^C$

(b)  $(C^C \cap A)^C$

(c)  $B \cap (A^C \cup C)$

9. How many distinct ways can the letters in the word HULLABALOO be arranged?

10. Suppose the weights of cats are normally distributed with an average weight of 8 pounds and a standard deviation of 1.75 pounds. What is the probability that a randomly selected cat weighs between 6 and 15 pounds?

11. Solve the following for  $a$ ,  $b$ ,  $c$ , and  $d$ .

$$4 \begin{bmatrix} -3 & 0 \\ 7 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 2 & -4 \\ 5 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & (c-2) \\ b & 4 \end{bmatrix}^T = \begin{bmatrix} (d+4) & -2 \\ 5 & a \end{bmatrix}$$

12. There is a fruit market that has 120 oranges, 500 cherries, and 200 apples. Of these, there are 4 rotten oranges, 100 rotten cherries, and 10 rotten apples. What's the probability that a customer will select 2 rotten oranges of 2 oranges he/she picked, 1 rotten apple of 1 apple he/she picked, and 30 rotten cherries of 40 cherries he/she picked?

13. It is known that 28% of a particular population enjoys eating seafood. From this population, 300 people are selected at random.

(a) What is the probability that exactly 80 people enjoy eating seafood?

(b) What is the probability that at least 75 people, but no more than 125 people, enjoy eating seafood?



14. You pay \$5.00 to play the following game. You have 2 chances to draw a ball from a bag. If you draw a white ball, you win nothing and if you draw a purple ball you win six dollars. In the bag there are 30 balls total - 20 are white and 10 are purple. After the first ball is drawn it is replaced before the next ball is drawn.
- (a) Find the expected net winnings of this game.
  - (b) Is this game fair? Why or why not?

15. You are given the following data:  $(0,0)$ ,  $(1,2)$ ,  $(3,5)$ ,  $(4,6)$ , and  $(6,9)$ , where  $x$ -values represent the number of items sold (in hundreds) and  $y$ -values represent the amount of profit made (in thousands of dollars).

(a) Find the least-squares line for the data.

(b) Is the line you found a good fit for the data? Why or why not?

(c) Use your line to predict the amount of profit made when 550 items are sold.

(d) Use your line to predict the amount of items which must be sold to generate a profit of \$32,500.

16. A child wants to build a block city. Each house requires 50 square blocks, 100 rectangular blocks, and 4 windows. Each store requires 50 square blocks, 125 rectangular blocks, and 8 windows. Each school requires 100 square blocks, 75 rectangular blocks, and 20 windows. If there are 5250 square blocks, 7375 rectangular blocks, and 880 windows, how many houses, stores, and schools can the child build if all of the materials are to be used?

17. Shade the portion of a Venn diagram that represents the set  $(A^C \cup B)^C \cap C^C$ .
18. Kathryn has a collection of 18 different Tweety Birds - 8 plush toys, 6 plastic figurines, and 4 porcelain figurines. She wants to arrange all of these on one shelf.
- (a) How many total arrangements exist?
- (b) How many total arrangements are possible if all Tweety Birds made out of the same material are grouped together?
19. Find the value of  $a$  if  $P(Z > a) = 0.6235$ , where  $Z$  is the standard normal random variable.

20. The quantity demanded for model Corvettes is 8000 if the price is \$20. If the price goes up to \$25, the quantity demanded goes down to 6000. A manufacturer will not market the models if the price drops below \$10. For every \$5 increase, he will produce 2000 more models.

(a) Find the demand function.

(b) Find the supply function.

(c) Find the equilibrium point.

21. Are the following matrices in reduced row-echelon form?

$$(a) \left[ \begin{array}{ccc|c} 1 & -2 & 0 & t \\ 0 & 1 & 3 & u \\ 0 & 0 & 1 & v \end{array} \right]$$

$$(b) \left[ \begin{array}{cccc|c} 0 & 0 & 1 & 0 & x \\ 1 & 0 & 0 & 0 & w \\ 0 & 0 & 0 & 0 & t \\ 0 & 0 & 1 & 0 & r \end{array} \right]$$

22. In 2002, 100 Aggies were surveyed concerning where they preferred to go on Friday nights.

- 35 liked Harry's and MadHatters
- 69 liked The Chicken
- 59 liked more than one of the three places
- 30 liked Harry's or MadHatters, but not The Chicken
- 15 liked Harry's and The Chicken, but not MadHatters
- 30 liked all three places
- 15 liked Harry's, but not The Chicken
- 1 liked to stay home and study for math!

- (a) Fill in an appropriate Venn diagram with the given information.
- (b) Express “the Aggies who liked only MadHatters” with set notation (set names, unions, intersections, and/or complements).

23. In a group of 200 people in Florida,  $\frac{3}{4}$  are Republicans and  $\frac{1}{4}$  are Democrats. In the 2000 presidential election, 95% of the Republicans voted for Bush, 4% voted for Gore, and the rest had unreadable ballots. On the other hand, 60% of the Democrats voted for Gore, 10% voted for Bush, and the rest had unreadable ballots. If an unreadable ballot is selected at random, what is the probability it was cast by
- (a) a Democrat?
  - (b) a Republican?

24. If the odds are 5 to 8 against an event occurring, what is the probability of

(a) the event occurring?

(b) the event not occurring?

25. Given  $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ ,  $B = \begin{bmatrix} 5 & 1 \\ 2 & 0 \\ 3 & 4 \end{bmatrix}$  and  $C = \begin{bmatrix} 6 & 1 & 3 & 4 \\ 7 & 5 & 1 & 0 \\ 2 & 1 & 8 & 1 \end{bmatrix}$

find:

(a)  $AB$

(b)  $BC$



26. A company makes 100 CDs with 10 of them being defective. You buy 8 from the factory directly.

(a) What is the probability that you get exactly 6 defective CDs?

(b) What is the probability that you get at least 1 defective CD?

27. Using the given Venn diagram, find the following:

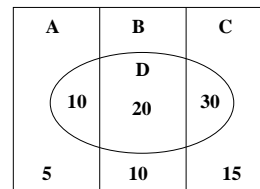
(a)  $P(D)$

(b)  $P(B \mid D)$

(c)  $P(D^C)$

(d)  $P(B \mid D^C)$

(e)  $P(B \cup D)$



28. Solve the following systems of equations.

$$\begin{aligned} \text{(a)} \quad & 3x - 5y = 6 \\ & -2x + 4y = -7 \\ & 2x - 4y = 6 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 2x - y - 3z = 3 \\ & 2x + 2y - z = 7 \\ & 3y + 2z = 4 \end{aligned}$$

29. SET UP the following linear programming problem, but do not solve.

A company produces two types of saddles, one english and one western. The english sells for \$350 and the western for \$600. The english saddle requires 5 units of leather, 12 hours assembly time, and 2 units of stitching. The western saddle requires 12 units of leather, 16 hours of assembly, and 4 units of stitching. The company only has 1100 units of leather, 32 eight-hour days for assembly, and 42 units of stitching. How many of each model should be produced in order for the company to maximize its revenue?

30. A poll is being conducted among readers of USA Today. Eight multiple choice questions are asked, each with 5 possible answers. In how many different ways can a reader complete the poll if exactly one response is given to each question?
31. All probabilities are between \_\_\_\_\_ and \_\_\_\_\_, inclusive.
32. A letter is picked at random from the word VACATION.
- (a) Write the uniform sample space of this experiment.
- (b) How many total events are associated with this experiment? Simple events?
- (c) Are the events “choosing a vowel” and “choosing a letter from the word CATS” mutually exclusive events? Independent events?
33. A pair of fair 6-sided dice are rolled. If the sum of the numbers which lands uppermost on the dice is 6 or 7, what is the probability that the number which lands uppermost on the second die is a 4 or 5?
34. The probability distribution of a random variable  $X$  is given. Compute the mean, median, variance and population standard deviation of  $X$ .

$X$	1	2	3	4
$P(X = x)$	0.4	0.3	0.2	0.1