

Exam I Review Problems

Fall 2011

Note: Not every topic is covered in this review.

Please also take a look at the previous Week-in-Reviews for more practice problems.

1. Using the following matrices, compute the given operations. If an operation is not possible, then explain why not.

$$A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 \\ 0 & 2 \\ 4 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & -2 & 3 \end{bmatrix}$$

- (a) $D + C =$
 - (b) $D - 3B =$
 - (c) $DC =$
 - (d) $DA =$
 - (e) $B + C^T =$
 - (f) $B^{-1} =$
 - (g) $A^{-1} =$
 - (h) $E^{-1} =$
 - (i) $5CAB =$
 - (j) $a_{12} - 3c_{32} + e_{22} =$
2. Solve for the variables x , y , z , and u . If this is not possible, then explain why not.

$$\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -2x & 0 \\ 3 & 4 \end{bmatrix} - 3 \begin{bmatrix} (y-1) & x \\ 4 & (2z+1) \end{bmatrix} = \begin{bmatrix} -7 & -2u \\ -10 & 8 \end{bmatrix}^T$$

3. Find the matrix K that makes the following true. If this is not possible, then explain why.

$$\begin{bmatrix} 0 & 8 & 1 \\ 7 & -6 & 0 \\ 3 & 0 & 1 \end{bmatrix} + \frac{1}{2}K \begin{bmatrix} 2 & 0 & 0 \\ 5 & 2 & 0 \\ 6 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 6 \\ 0 & 1 & 4 \\ 3 & 7 & 0 \end{bmatrix}$$

4. Use the following matrices to determine which of the products below make sense. Bolli Bros. and Pizza Nut are two restaurants which each sell pizza rolls (in batches of 6 rolls), calzones and orders of cheese sticks. The prep and cook times for each food item is given, as well as the number of items two school districts will order.

$$A = \begin{matrix} & \begin{matrix} \text{BolliBros.} & \text{PizzaNut} \end{matrix} \\ \begin{matrix} \text{PizzaRolls} \\ \text{Calzone} \\ \text{CheeseSticks} \end{matrix} & \begin{bmatrix} \$5.95 & \$8.00 \\ \$6.95 & \$9.99 \\ \$3.95 & \$6.99 \end{bmatrix} \end{matrix}$$

$$B = \begin{matrix} & \begin{matrix} \text{PizzaRolls} & \text{Calzone} & \text{CheeseSticks} \end{matrix} \\ \begin{matrix} \text{PrepTime(Min)} \\ \text{CookTime(Min)} \end{matrix} & \begin{bmatrix} 5 & 3 & 2 \\ 6 & 7 & 6 \end{bmatrix} \end{matrix}$$

$$C = \begin{matrix} & \begin{matrix} \text{BISD} & \text{CSISD} \end{matrix} \\ \begin{matrix} \text{PizzaRolls} \\ \text{Calzone} \\ \text{CheeseSticks} \end{matrix} & \begin{bmatrix} 100 & 85 \\ 75 & 100 \\ 50 & 60 \end{bmatrix} \end{matrix}$$

- (a) AB
 (b) BA
 (c) BC
5. An investor has \$50,000 he invests into three accounts yielding 2%, 8%, and 4% interest/year, respectively. If he earns a total of \$4,000 interest in one year, and if he invests twice as much at 4% as he does at 2%, how much does he invest in each account?
- (a) Set up a system of equations representing this problem.
 (b) Write your system of equations as a matrix equation, $AX = B$.
 (c) Solve your matrix equation by using matrix inverses.
 (d) Solve your system of equations by using an augmented matrix.
6. Find the equation of the line that passes through the y -intercept of the line $4x + 2y = 7$ and also passes through the x -intercept of the line $y = 9x + 8$.
7. Find the value of m that makes the following lines parallel. Verify that you will have two distinct lines.

$$3x - 7y = -10$$

$$8x + my = 15$$

8. A sailor buys a yacht for \$1,000,000. After 10 years it is worth \$800,000.
- (a) Assuming the yacht is depreciating linearly, find the value of the yacht as a function of the number of years the sailor has owned it.
 (b) Assuming a scrap value of \$0, how long will the yacht have value?

9. A company making blankets incurs a total cost of \$3375 when producing 250 blankets and a total cost of \$3750 when producing 300 blankets. The company sells the blankets for \$20 each.

- (a) What are the fixed costs for this company?
- (b) Find the company's linear profit function.
- (c) What is the break-even point for this company?

10. Given two functions $3x - 11p + 45 = 0$ and $2x + 7p - 56 = 0$, where x represents the number of items made/sold and p represents the price of the item, what are the equilibrium price and equilibrium quantity which would satisfy both consumers and producers?

11. The following data are representative of information in *Energy Policy*, March 1983. The data represents carbon dioxide (CO₂) emissions from coal-fired boilers (in units of 1000 tons) over a period of years between 1965 and 1977. For this problem let the time start with zero in 1965.(i.e. 1965 is zero)

Year(x)	0	5	8	9	10	11	12
CO ₂ emission (y)	910	680	520	450	370	380	340

- (a) Determine the equation of the least-squares line for this data. Round all coefficients/constants to 4 decimal places.
- (b) Is the line you found a good fit for the data? Why or why not?
- (c) Predict the carbon dioxide emissions in the year 1972, to the nearest ton.
- (d) In what year would we expect to have carbon dioxide emissions of 200,000 tons?

12. Are the following augmented matrices in reduced row-echelon form?

If YES, how many solutions will the system have?

If NO, explain which property is violated.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 5 \end{array} \right]$

(b) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$

(c) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & 1 & 0 & 15 \\ 0 & 0 & 0 & 0 \end{array} \right]$

(d) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & 1 & 1 \end{array} \right]$

(e) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 8 \\ 0 & 1 & 0 & 9 \end{array} \right]$

(f) $\left[\begin{array}{cc|c} 1 & 3 & 100 \\ 0 & -1 & 200 \end{array} \right]$

(g) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 18 \\ 0 & 0 & 0 & 0 \end{array} \right]$

13. (a) Use the Gauss-Jordan Elimination Method (showing all row operations and intermediate matrices) to find the equivalent reduced row-echelon form for the following: $\left[\begin{array}{cc|c} 2 & 15 & 50 \\ -3 & 7 & 20 \end{array} \right]$

(b) Check your answer to part (a) by using *rref* on your calculator.

14. Solve the following systems of equations.

(a) $-2x + 5y - 3z = 30$
 $x - y - 2z = 50$
 $x - 7y + 12z = -210$

(b) $20x - 15y = 70$
 $x - y = 2$
 $-8x + 3y = -30$

15. A company makes two types of toys: zappers and wompers. Each zapper takes 5 oz. of plastic and 4 minutes to manufacture. Each womper takes 3 oz. of plastic and 2 minutes to manufacture. There is available 5250 oz. of plastic and 60 hours of manufacturing time. If the company makes a total of 1650 toys and all resources are to be used, determine how many of each toy the company can make.