Mathematical and computational aspects of chemical enhanced oil recovery process

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Abstract

In this talk, we give a brief overview of some of the fundamental theoretical and computational problems in EOR technology that are of interest. We present some recent fundamental results on some of these problems obtained by the speaker and his collaborators. We also present a new efficient, high order accurate high performance numerical method for solving a coupled highly nonlinear system of partial differential equations arising in the context of the fractional flow model of Alkali-Surfactant-Polymer flooding of oil reservoirs. The method is based on the use of an extended FEM elliptic solver, an implicit-time linearized FDM, and level set formulation. We will present numerical results using this method. The research reported here has been made possible by a grant (NPRP 08-777-1-141) from the Qatar National Research Fund (a member of The Qatar Foundation). The statements made herein are solely responsibility of the authors.

Thermodynamic Interpretation of Enhanced Oil Recovery: A Focus on Chemical Flooding

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Numerous, and gradually increasing umber of papers prove that wider and routine application of advanced chemical methods will become inevitable part of production methods in the future, and emblematic authors postulate that all intensive technology are partly or fully based on chemical mechanisms. Since the present comprehension and interpretation of recovery methods seems to be obsolete and patchy, new interdisciplinary approach is necessary to step forward smart flooding and stimulation technologies, and chemistry, more exactly oilfield chemistry will play a fundamental role to realize those goals. Oilfield chemistry is a multidisciplinary branch of sciences integrating the knowledge of reservoir engineering production engineering chemistry and chemical engineering and many more. Mission of oilfield chemistry is to increase the recovery efficiency up to a possible ultimate limit making available the explored hydrocarbon resources and thus, meeting the global demand of humankind as long as possible. This new branch of engineering science deals with special and unique feature in respect to the source of problems, scientific approach, development, and application of technologies. Oilfield chemistry is focusing on processes taking place

- at high pressure and temperature,
- in multiphase (oil/water/gas/rock) systems,
- in heterogeneous porous and fractured systems, and
- under dynamic conditions.

The oilfield chemistry already became a core element of exploration (geochemistry), drilling (mud chemistry), stimulation (fracturing, acidizing, bottomhole clean-up), productivity and injectivity improvement (chemical selective shutoff and conformance treatment), mitigation of formation damage (scale and corrosion chemistry, paraffin and asphaltene removal), IOR/EOR chemistry (chemical, gas and thermal and microbial methods), water management (water chemistry), phase separation and enrichment in surface facilities (emulsion and suspension breaking). In addition, chemistry has fundamental role in recovery of unconventional hydrocarbons by leaching and extraction of heavy oils and bitumens, tapping gas from tight sand shale reservoirs and basin concentrated gas accumulation, methane production from coal seams, chemical decomposition of hydrates, etc. The paper to be presented will focus on chemical EOR methods using thermodynamic interpretation of processes. It will be firmly shown that the diverse physical and chemical factors influencing the displacement phenomena don't form a common and additive parameter, which can be use as a universal parameter characterizing the displacement process, and among others, the recovery efficiency. In contrast, I will be clearly stated that the universal basis of composite mechanism might be the additive activation energy of side processes. The main initiative is the recognition of the fact that Some IOR/EOR methods do not decrease, but rather increase the thermodynamic threshold potential (energy demand) compared to the conventional, low efficiency recovery process. Consequently, they represent thermodynamically *divergent* solutions even though these methods might be beneficial from certain engineering point of view.

That principle also provides a reliable basis for a new classification of chemical EOR. Namely, depending on the resultant (cumulative) activation energy of contributing processes of mechanism, the EOR methods can be convergent, divergent and mixed (convergent/divergent or divergent/convergent) flooding methods. For instance, a typical convergent method is the gas injection or thermal recovery technology, the polymer flooding representing the divergent method; meanwhile the surfactant/polymer flooding can be classified as a mixed procedure. Thus, the presentation focusing on the state-of-the-art of laboratory and field efforts, it will predict the importance of chemicals and chemical technologies in the coming years. In addition, the fundamental message is that the art of chemical EOR design should be a meticulous balancing between the divergent and convergent segments of hybrid flooding methods with the aim at keeping the total activation energy at minimum level.

Based on the novel approach of EOR methods, the following general conclusions will be disseminated:

- Chemistry is an unquestionable part of reservoir and production engineering;
- The role of IOR/EOR technologies is gradually increasing and the chemical methods will form a mainstay of production engineering; the potential of chemical methods are, however, underestimated at present;
- The oil price will probably increase in the future, and that trend will also characterize the coming years opening new perspectives to the chemical methods;
- The petroleum engineering and the relevant scientific areas got to the phase of "revolutionary science". Namely, the old ideas, approaches must be replaced by new paradigms;
- Building globally knowledge based, and cooperative society and the R&D activity in the petroleum industry should play a key role in global energy supply and sustainable development of the mankind.

Fundamental message of the presentation is that "Age of Chemistry" is ahead of us, and chemistry will have a strong and beneficial effect on progress of the upstream sector of the petroleum industry.

Microbial Enhanced Oil Recovery Processes

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Abstract

Enhanced oil recovery, or EOR can be defined as advanced processes used to further increase oil recovery beyond secondary recovery by conventional methods in a reservoir. Enhanced oil recovery processes include all methods that use external sources of energy and/or materials to recover oil that cannot be produced economically by conventional means. The widely used EOR methods include thermal (injection of thermal energy such us steam flooding), chemical (injection of interfacial-active components such as surfactants, alkalis, polymers, and chemical blends), and Miscible or Solvent Injection such as hydrocarbon miscible and carbon dioxide allowing a decrease of the oil viscosity.

Microbial-enhanced oil recovery (MEOR) could fall in any of the aforementioned methods, but some of the mechanisms involved are not fully understood. MEOR can be defined as a tertiary recovery process where bacteria and their metabolic byproducts are used for oil mobilization in a reservoir. Numerous MEOR mechanisms involve (1) interfacial tension reduction due to the microbial production of amphiphilic compounds often called bio-surfactants, which reduce the interfacial tension between immiscible phases, (2) wettability change due to biofilms formation with wetting properties significantly different from that of the existing reservoir rock, (3) bio-clogging due to biomass formation, which can clog preferential flow path and increase a reservoir's sweep efficiently by diverting flow to alternate paths, and (4) biogenic gas generation such as methane and carbon dioxide, which can increase the pore pressure and/or dissolve into the oil phase, thus reducing viscosity and swelling the oil.

Generally, MEOR operations involve injection of nutrients, typically together with cultivated-exogenous microbes, into the reservoir. The injected nutrients stimulate microbial activity leading to biomass production and byproducts formation (e.g. gases, bio-surfactants and bio-polymers). Modification of in-reservoir environment and the oil itself help to mobilize the residual oil toward production wells. Compared with other EOR methods such as chemical EOR, MEOR is considered to be more cost effective and environmentally friendly, since the nutrients (and microbes) are relatively less expensive than the chemicals and biodegradable. Furthermore, indigenous bacteria, which are already adapted to the in-reservoir conditions (temperature, pressure, salinity, redox, pH…) may be stimulated by adequate nutrients injection making the process economically more effective. However, indigenous sulfate-reducing bacteria, often found in deep, hot and high-pressure reservoirs may cause their souring via hydrogen sulfide production.

Several field studies confirmed the potential success of MEOR with a net oil production increase approximating 40% in certain cases. However, a better understanding of the MEOR processes and mechanisms from an engineering standpoint based on economics, applicability and performance is the key to further improve the process efficiency.

Multicomponent polymer flooding in two dimensional oil reservoir simulation

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Abstract

We propose a high resolution finite volume scheme for a $(m+1)\times(m+1)$ system of nonstrictly hyperbolic conservation law which models multicomponent polymer flooding in enhanced oil-recovery process in two dimensions. Here the state space consists of saturation of the aqueous space s and concentration $(c_1, c_2, ..., c_m)$ of different m polymers dissolved in the aqueous space. It is well known that in the heterogeneous media, that is when the permeability $K(x)$ is discontinuous, fingering instability will develop and which results in an inefficient oil-recovery. As the concentration c increases, viscosity of water increases and the fingering effects reduces which leads to an efficient oil-recovery. In the presence of the concentration c, the system becomes coupled and non-strictly hyperbolic. Also in the presence of gravity the flux functions need not be monotone and hence construction of upwind schemes are difficult. Most often numerical methods requires the calculation of eigenvalues and eigenvectors of the Jacobian matrix of the system. Here by using the idea of discontinuous flux we reduce the system to an uncoupled scalar equations with discontinuous coefficients. Next we study each scalar equation by using the idea of discontinuous flux. This approach does not require detailed information about the eigenstructure of the full system. First, in one space dimension for the scalar conservation law, we show how to extend the Godunov scheme to the fluxes which are discontiuous in the space variable. By using the special property of the Buckley-Leverett equation, we show how the idea of discontinuous flux can be applied to the case of the above system. Next, for the case $m = 1$ and in the one space dimension, we compare the Godunov numerical scheme obtained from discontinuous flux idea, called DFLU scheme with the exact Godunov scheme of the 2×2 system and show the efficiency of the DFLU scheme. We compare also numerical results with Upstream mobility scheme, Force and Lax-friedrichs schemes. High order accurate scheme is constructed by introducing slope limiter in two dimensional space variable and a strong stability preserving Runge-Kutta scheme in the time variable. The performance of the numerical scheme is presented in various situations by choosing a heavily heterogeneous hard rock type medium. Also the significance of dissolving polymers in aqueous phase is presented. This talk is mainly taken from my joint works with Adimurthi, Jerome Jaffre, Sudarshan Kumar and Praveen.

Lax- Friedrichs Scheme for conservation laws with discontinuous flux

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Abstract

Conservation laws with discontinuous flux arises in many physical problems such as extraction of oil, sedimentation problems etc. Basically it is a model based on two phase flow problem. This has been studied extensively from last few years. Existence and unicity of solutions has been studied with an extra entropy condition at the interphase known as (A, B) interphase condition through Godunov and Enquist Osher schemes. It was not obvious to extend to Lax Friedrichs scheme due to lack of consistency of the scheme. Here we overcome these difficulties and provide an appropriate extension and show the convergence. This is the joint work with Veerappa Gowda, Jerome Jaffre and Rajib Dutta.

Saffman-Taylor instability for an Oldroyd-B fluid

by

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This following abstract is based on current ongoing joint work with Prabir Daripa of Texas A&M University.

In chemical enhanced oil recovery and drilling technologies, use of complex fluids is very common. The displacement process of such a complex fluid by another simple or complex fluid in a porous mediuum plays a very important role and is not very well-understood. The complexity of interfacial instability in such displacement processes depend strongly on the rheological properties of such complex fluids and can be different from one complex fluid to another complex fluid. There have been some studies of such flows in recent years. We attempt to contribute to this wealth of knowledge in this area by undertaking studies of interfacial instabilities in such complex fluids.

In this talk, we extend the classical Saffman-Taylor instability to non-Newtonian case. In particular, we consider an incompressible Oldroyd-B fluid displaced by air in a Hele-Shaw cell. Fluid flow in a Hele-Shaw cell has proven over the years as a decent model of porous media flow, in particular when the medium is homogeneous. Due to the very complicated constitutive relations between the extra-stress and strain-rate tensors, the perturbation equations are derived from the full flow equations. The depth-average procedure is performed only in the Laplace law on interface as a final step. A scaling procedure is used due to the particular flow geometry - a very thin Hele-Shaw cell. As a consequence, some terms in the constitutive relations can be neglected. The lubrication approximation is also used by neglecting the vertical component of the velocity. This way, we obtain an explicit expression of one of the components (τ_{11}) of the extra-stress perturbations tensor in terms of the horizontal velocity perturbations. The main result is an explicit formula for the growth constant (in time) of perturbations, given by a ratio. In this formula, a term depending on the relaxation and retardation (time) constants appears in the denominator of the ratio. When these two constants tend to zero, we recover the Saffman-Taylor result for a Newtonian fluid displaced by air. The analysis will be presented and its relevance to EOR will be discussed.

Physically realistic reservoir simulation of fractured or structurally complex reservoirs

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For production optimization and forecasting of EOR performance, reservoir simulation must be capable of expressing emergent behavior arising from the nonlinear interplay of gravitational, viscous and capillary forces. In fractured or structurally complex reservoirs, these processes simultaneously affect hydrocarbon saturation across a wide range of length scales. There is mounting evidence, that present-day commercial simulators fail to achieve this objective because of three principle reasons: 1) they cannot resolve high length-over-width ratio, highly permeable faults or fracture corridors because their shape and complex intersection geometry cannot be captured with structured grids; 2) they are based on a “point-property” finite-difference discretization that blurs material interfaces by averaging flow terms across them, 3) their default space - time discretization methods are of lowest possible order and cannot express fingering and coarsening instabilities. This leads to artificially stable flow and over-optimistic estimates of sweep. Deficiencies 1 to 3 imply that, when these simulators are used to create reservoir response surfaces and Pareto charts derived from experimental design studies, the results are misleading with regard to the influence of reservoir parameters and production measures on recovery. Regarding the simulation of naturally fractured reservoirs, commony used dual continua approximations further detach simulation predictions from reservoir reality because this conceptual approach cannot capture the geometrical characteristics of natural fractures, including their multi-model aperture statistics. The dual continuum approach also artificially separates the fractures from the rock matrix and treats viscous, gravitational, and capillary displacements independently from each other although laboratory and field evidence shows that they are strongly coupled (Matthai and Bazrafkan, 2011).

This presentation will show that adaptive space-time discretization of fractured and-or structurally complex reservoirs and production processes is key for the construction of realistic reservoir models, expressing emerging behaviour and allowing response forecasts for dynamic (reservoir) systems. It will be shown that space-time adaptivity is decisive for computational efficiency. Embracing the additional concept of goal-based simulation, striving for an accuracy that can be predefined by the user, spatial discretization errors need to be dealt with using an error metric, and - in the discretization of time - stepping needs to adapt to the wide range of flow velocities in physically realistic reservoir models.

This presentation provides first examples of how space-time adaptivity can be applied to achieve more realistic simulations of naturally fractured reservoirs, leading the way to goal-based simulation. For this purpose, examples are presented that stem from a novel characterization, modeling and simulation workflow for naturally fractured reservoirs (Fig. 1), that is available to operators through an academic-industry partnership.

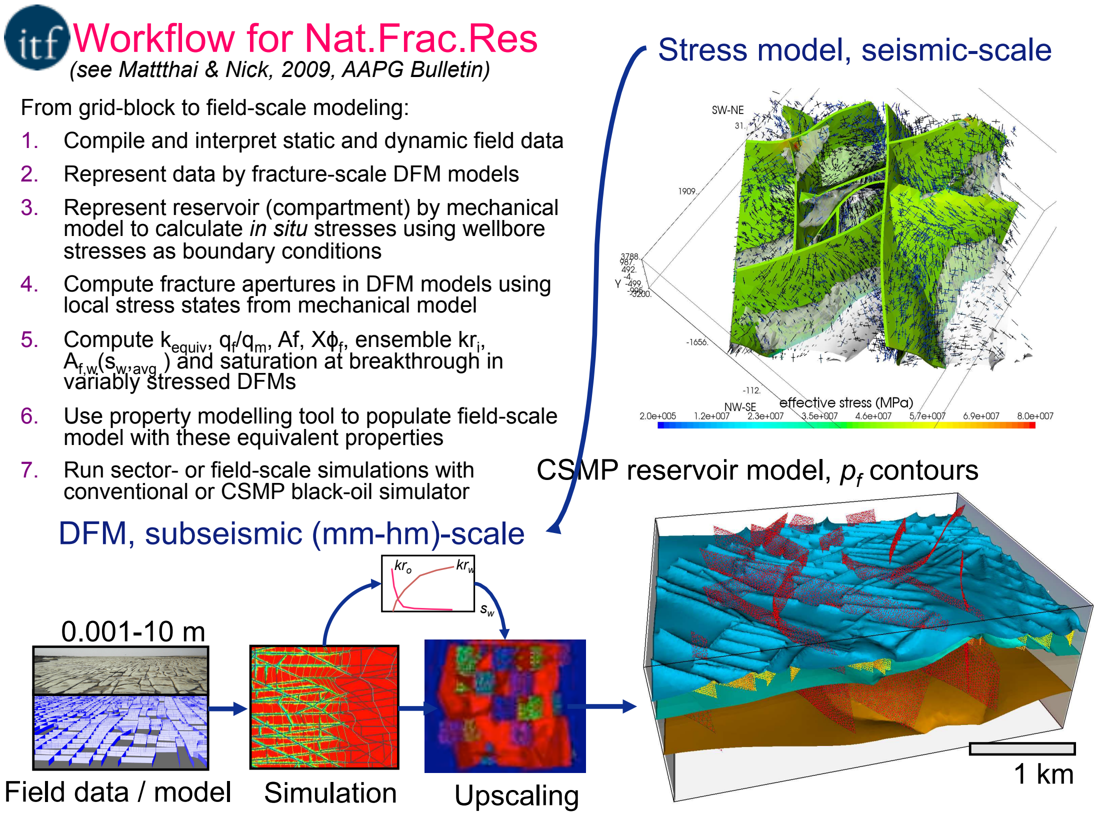

Figure 1: Novel workflow for the numerical simulation based upscaling of multiphase flow in naturally fractured reservoirs (Matthai and Nick, 2009).

References

Bazrafkan, S. and Matthai, S. K., Issues with Dual Continua Formulations. SPE Applied Technology Workshop, 28-30 November 2011, Beach Rotana Hotel, Abu Dhabi, UAE.

Matthäi, S. K. and Nick, H. [2009] Upscaling two-phase flow in naturally fractured reservoirs. AAPG Bulletin 93:11, 1-12.

Matthäi, S.K., Mezentsev, A., and Belayneh, M. [2007] Finite-Element Node-Centered Finite-Volume experiments with fractured rock represented by unstructured hybrid element meshes. SPE Reservoir Evaluation & Engineering, 10:6, 740-756.

Miscible Gas Injection Challenges: Optimum Gas Mixture to Maximize Oil Recovery from Medium/Light Crude Oil Reservoirs

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For the longest time, single miscible gas injection has been used as EOR method utilized to improve oil recovery from different medium/light crude oil reservoirs. Only recently it became evident that none of these solvents have the necessary characteristics required to boost RF economically. Miscible gas injection is an EOR method deployed to increase oil production. Combinations of Carbon dioxide with other gases as miscible solvents are emphasized in this study. Emphasis is placed on identifying CO_2/solvent mixtures with reduced MMP to achieve miscibility at reasonable injection pressures. Two targeted crude oils (Oil 1 and Oil 2) from two carbonate Middle East reservoirs are utilized.

The minimum miscibility pressure (MMP) of the targeted oils with mixtures of N_2, CH_4, C_2H_6, and HC rich gas of varying composition with CO_2 injection gas are evaluated through simulation procedure. Cell to Cell and Semi-analytical (key tie lines) methods are applied using CMG simulator. Results show that miscibility is predicted to occur with multiple contact miscibility (MCM): vaporization and/or condensation mechanisms. The increase of C_2H_6 concentration in the CO_2 injected gas reduced MMPs for targeted Oil 1 by 100 psi/10 mol%. However, N_2, CH_4 and HC rich gas increments in CO_2 injected gas increased the MMPs for targeted Oil 1. MMP was observed to be 2300 psi for pure ethane with Oil 1. In addition, MMPs for targeted oils with N_2/ C_2H_6 and N2/ CH_4 injected gas mixtures are assessed. This study can open possibilities for future enriching of CO_2 and N_2 miscible injection to improve miscibility and recovery of oil.

As part of the first phase of this study, a compositional simulation model was built and run by using the CMG WinProp module to predict the MMPs of CO2 miscible flooding enriched with N_2 and other hydrocarbon gases. The MMP for these oils could increase unfavorably as the N2 and/or CH4 concentration increased in the CO2 stream. The MMP changes as the type and concentration of enrichment in the injected CO2 stream change. This study concludes that none of the solvent gases, normally used for miscible injection, can achieve the optimum results. It is necessary to continue the efforts of experimental and analytical investigation to search for the "magic" solvent mixture that gives best recovery results.

Forward and inverse problems in multiphase flow through porous media

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Abstract: Mathematical models of fluid flow problems in porous media result in non-linear (system of) Partial Differential Equations (PDEs). Due to complexity of the PDEs and/or of the application domains, analytical solutions to these equations do not exist in general. One possible alternative is to look for approximate numerical solutions. Therefore, PDE-simulation is widespread in scientific and engineering applications in these fields. As PDE-solvers mature, there is increasing interest in industry and academia in solving optimization problems governed by those PDEs. These optimization problems are quite challenging since the size and the complexity of the discretized PDEs often pose significant difficulty for the contemporary optimization methods.

This talk will focus on numerical methods for such problems. Special emphasis will be given to application areas in Environmental Engineering. In this class of problems, the PDEs involved are non-stationary and the CFD is quite expensive in terms of CPU and memory requirements, especially when adjoint based optimization methods are applied for the solution of the optimization problems. For these problems a sensitivity-based multiple-shooting method has been developed. This method is quite efficient since the optimality is achieved simultaneously with the feasibility of the state equations. Some of the results, obtained using this method, will be presented as well.

Finite Volume–Nonconforming Finite Element For Two Compressible And Immiscible Flow In Porous Media

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1 Introduction

The cell-centered finite volume method with an upwind discretization of the convection term ensures the stability and is extremely robust and have been used in industry because they are cheap, simple to code and robust. However, standard finite volume schemes do not permit to handle anisotropic diffusion on general meshes see e.g. [5]. On the other hand finite element method allows a very simple discretization of the diffusion term with a full tensor and does not impose any restrictions on the meshes, they were used a lot for the discretization of a degenerate parabolic problems modeling of contaminant transport in porous media. However, it is well-known that numerical instabilities may arise in the convection-dominated case. To avoid these instabilities, the theoretical analysis of the combined finite volume/finite element method has been carried out for the case of a degenerate parabolic problems with a full diffusion tensors. The combined finite volume–conforming finite element method proposed and studied by Debiez and al [4] or Feistauer and al [7] for fluid mechanics equations. This ideas is extended by Voharlik and al [9] for the degenerate parabolic problems.

In the present work, we consider a two-phase flow model where the fluids are immiscible. The medium is saturated by a two compressible phase flows. The model is treated without simplified assumptions on the density of each phase, we consider that the density of each phase depends on its corresponding pressure. It is well known that equations arising from multiphase flow in porous media are degenerated. The first type of degeneracy derives from the behavior of relative permeability of each phase which vanishes when his saturation goes to zero. The second type of degeneracy is due to the time derivative term when the saturation of each phase vanishes.

For homogeneous and isotropic diffusion tensor, the scheme proposed in [8] and in [1] consists in an implicit finite volume method together with a phase-by-phase upstream scheme which satisfies industrial constraints of robustness. The scheme is constructed on orthogonal mesh. The authors show that the proposed scheme satisfy the maximum principle for the saturation and a discrete energy estimate on the velocity of each phase.

Here, we are interested by inhomogeneous and anisotropic diffusion tensor. This work deals with construction and convergence analysis of a combined finite volume–nonconforming finite element for two compressible and immiscible flow in porous media without simplified assumptions on the state law of the density of each phase. We consider a triangulation of the space domain consisting of simplices (triangles in space dimension two and tetrahedra in space dimension three). We next construct a dual mesh where the dual volumes are associated with the sides (edges or faces). The diffusion term, which can be anisotropic and heterogeneous, is discretized by piecewise linear nonconforming triangular finite elements. The other terms are discretized by means of a cell-centered finite volume scheme on a dual mesh, where the dual volumes are constructed around the sides of the original mesh, hence we obtain the combined scheme. To ensures the stability, the relative permeability of each phase is decentred according the sign of the velocity at the dual interface. This technique ensures the validity of the discrete maximum principle for the saturation in the case where all transmissibilities are non-negative.

We prove the convergence of the scheme, only supposing the shape regularity condition for the original mesh. In addition, we present some numerical tests in 2D on an unstructured mesh for water-gas flows in porous media.

References

[1] M. Bendahmane, Z. Khalil, M. Saad, *Convergence of a finite volume scheme for gas-water flow in a multi-dimensional porous medium.* Math. Models Methods Appl. Sci. DOI: 10.1142/S0218202513500498. Pages 1–41, 2013.

[2] F. Caro and B. Saad and M. Saad, *Study of degenerate parabolic system modeling the hydrogen displacement in a nuclear waste repository,* accepted DCDS-S (2013).

[3] F. Caro and B. Saad and M. Saad, *Two-component two-compressible flow in a porous medium,* Acta Applicandae Mathematicae, 117,15-46 (2012).

[4] C. Debiez, A. Dervieux, K. Mer, and B. Nkonga. *Computation of unsteady flows with mixed finite volume/finite element upwind methods.* Internat. J. Numer. Methods Fluids, 27:193–206, 1998.

[5] R. Eymard, T. Gallouet, and R. Herbin, *Finite Volume Methods.* Handbook of Numerical Analysis, North-Holland, Amsterdam, 2000.

[6] R. Eymard and V. Schleper, *Study of a numerical scheme for miscible two-phase flow in porous media,* hal-00741425 (2012).

[7] M. Feistauer, J. Felcman, and M. Lukacova, *On the convergence of a combined finite volume finite element method for nonlinear convection.* Numer. Methods Partial Differential Equations, 13(2):163–190, 1997.

[8] B. Saad, M. Saad *Study of full implicit petroleum engineering finite volume scheme for compressible two phase flow in porous media.* SIAM J. Num. Ana.51(1), pp. 716-741, 2013.

[9] M. Vohralik, R. Eymard, D. Hilhorst, *A combined finite volume–nonconforming/mixed–hybrid finite element scheme for degenerate parabolic problems.* Numer. Math., 105:73–131, 2006.

Parallel Adaptive Simulation of Processes from Science and Engineering

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Numerical simulation has become one of the major topics in Computational Science. To promote modelling and simulation of complex problems new strategies are needed allowing for the solution of large, complex model systems. Crucial issues for such strategies are reliability, efficiency, robustness, usability, and versatility.

After discussing the needs of large-scale simulation we point out basic simulation strategies such as adaptivity, parallelism and multigrid solvers. To allow adaptive, parallel computations the load balancing problem for dynamically chaniging grids has to be solved efficiently by fast heuristics. These strategies are combined in the simulation system UG ("Unstructured Grids") being presented in the following.

In the second part of the seminar we show the performance and efficiency of this strategy in various applications. In particular large scale parallel computations of density-driven groundwater flow is discussed in more detail. We further present models for biogas production and optimization.

Mathematical analysis of a three-layer fluid flow in a Hele-Shaw cell and porous medium

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Abstract

The following abstract is based on joint work with Prabir Daripa of Texas A&M University at College Station and Rodrigo Meneses of Escuela de Ingeniería Civil, Facultad de Ingeniería, Universidad de Valparaíso.

The displacement of three immiscible fluids separated from each other by distinct interfaces in a Hele-Shaw cell is an idealized model of one of the chemical enhanced oil recovery processes in homogeneous porous medium. The middle-layer fluid has a viscous profile with non-negative gradient in the direction of displacement which can be thought of an aqueous phase containing polymer with a non-negative concentration profile. This middle-layer fluid is displacing and is also displaced by fluids of constant viscosities. Fluid flow in this three-layer set-up is governed by (a) the conservation of mass, (b) the Darcy's law, and (c) the passive advection of viscosity. A detailed discussion of this model is given in Daripa [1] and some of the references cited therein. As discussed in these references, the governing system admits a constant displacement velocity (U) of the entire set-up with planar interfaces between the fluid layers. Perturbing this basic solution and linearizing the resulting system in a frame moving with velocity U gives a second order Sturm-Liouville problem (SLP) for the initial amplitude of the perturbation. The coefficients in the SLP depends on the jumps in viscosity at two interfaces, the wave number of perturbation, the displacement velocity U of the basic solution, and the growth rate (σ) of perturbation. The interfacial conditions depend linearly on the spectral parameter ($\lambda = 1/\sigma$), the interfacial tensions, the viscosities of the extreme layer fluids, the viscosity jumps at two interfaces, and depend nonlinearly on the wave number (k) of perturbation. In this sense, it is a regular but a non-standard SLP.

In this work, we analyze this SLP using a semi-analytical technique when the viscous profile of the middle layer is a linear function of the moving coordinate. In particular, this SLP is first reduced to Kummer's equation using a transformation and change of variables. The solution of the resulting equation is represented in terms of the Kummer and the Tricomi confluent hypergeometric functions. This in turn enables us to obtain an implicit complicated

nonlinear equation in the growth rate which can be solved numerically to obtain dispersion curves. This can then be compared with the dispersion curves obtained directly from solving the SLP numerically. These results and much more related to this equation will be presented in this talk. The relevance of this study to EOR will be discussed.

[1] P. Daripa, *Hydrodynamic stability of multi-layer hele-shaw flows*, Journal of Statistical Mechanics, P12005, (2008), 32 pages; DOI: 10.1088/I742-5468/2008/I2/P12005

Keywords: Hele-Shaw flow, non-standard Sturm-Liouville problem, growth rate, confluent hypergeometric functions, dispersion relation.

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