2 Exercises

In Exercises 1–5, write the given vector by using the standard basis vectors for $\mathbf{R}^2$ *and* $\mathbf{R}^3$.

1. $(2, 4)$ **2.** $(9, -6)$ **3.** $(3, \pi, -7)$

4. $(-1, 2, 5)$ **5.** $(2, 4, 0)$

In Exercises 6–10, write the given vector without using the standard basis notation.

6. $\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

7. $9\mathbf{i} - 2\mathbf{j} + \sqrt{2}\,\mathbf{k}$

8. $-3(2\mathbf{i} - 7\mathbf{k})$

9. $\pi\mathbf{i} - \mathbf{j}$ (Consider this to be a vector in $\mathbf{R}^2$.)

10. $\pi\mathbf{i} - \mathbf{j}$ (Consider this to be a vector in $\mathbf{R}^3$.)

11. Let $\mathbf{a}_1 = (1, 1)$ and $\mathbf{a}_2 = (1, -1)$.

(a) Write the vector $\mathbf{b} = (3, 1)$ as $c_1\mathbf{a}_1 + c_2\mathbf{a}_2$, where c_1 and c_2 are appropriate scalars.

(b) Repeat part (a) for the vector $\mathbf{b} = (3, -5)$.

(c) Show that *any* vector $\mathbf{b} = (b_1, b_2)$ in $\mathbf{R}^2$ may be written in the form $c_1\mathbf{a}_1 + c_2\mathbf{a}_2$ for appropriate choices of the scalars c_1, c_2. (This shows that $\mathbf{a}_1$ and $\mathbf{a}_2$ form a basis for $\mathbf{R}^2$ that can be used instead of $\mathbf{i}$ and $\mathbf{j}$.)

12. Let $\mathbf{a}_1 = (1, 0, -1)$, $\mathbf{a}_2 = (0, 1, 0)$, and $\mathbf{a}_3 = (1, 1, -1)$.

(a) Find scalars c_1, c_2, c_3, so as to write the vector $\mathbf{b} = (5, 6, -5)$ as $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + c_3\mathbf{a}_3$.

(b) Try to repeat part (a) for the vector $\mathbf{b} = (2, 3, 4)$. What happens?

(c) Can the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ be used as a basis for $\mathbf{R}^3$, instead of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$? Why or why not?

In Exercises 13–18, give a set of parametric equations for the lines so described.

13. The line in $\mathbf{R}^3$ through the point $(2, -1, 5)$ that is parallel to the vector $\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$.

14. The line in $\mathbf{R}^3$ through the point $(12, -2, 0)$ that is parallel to the vector $5\mathbf{i} - 12\mathbf{j} + \mathbf{k}$.

15. The line in $\mathbf{R}^2$ through the point $(2, -1)$ that is parallel to the vector $\mathbf{i} - 7\mathbf{j}$.

16. The line in $\mathbf{R}^3$ through the points $(2, 1, 2)$ and $(3, -1, 5)$.

17. The line in $\mathbf{R}^3$ through the points $(1, 4, 5)$ and $(2, 4, -1)$.

18. The line in $\mathbf{R}^2$ through the points $(8, 5)$ and $(1, 7)$.

19. Write a set of parametric equations for the line in $\mathbf{R}^4$ through the point $(1, 2, 0, 4)$ and parallel to the vector $(-2, 5, 3, 7)$.

20. Write a set of parametric equations for the line in $\mathbf{R}^5$ through the points $(9, \pi, -1, 5, 2)$ and $(-1, 1, \sqrt{2}, 7, 1)$.

21. (a) Write a set of parametric equations for the line in $\mathbf{R}^3$ through the point $(-1, 7, 3)$ and parallel to the vector $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$.

(b) Write a set of parametric equations for the line through the points $(5, -3, 4)$ and $(0, 1, 9)$.

(c) Write different (but equally correct) sets of equations for parts (a) and (b).

(d) Find the symmetric forms of your answers in (a)–(c).

22. Give a symmetric form for the line having parametric equations $x = 5 - 2t$, $y = 3t + 1$, $z = 6t - 4$.

23. Give a symmetric form for the line having parametric equations $x = t + 7$, $y = 3t - 9$, $z = 6 - 8t$.

24. A certain line in $\mathbf{R}^3$ has symmetric form

$$\frac{x-2}{5} = \frac{y-3}{-2} = \frac{z+1}{4}.$$

Write a set of parametric equations for this line.

25. Give a set of parametric equations for the line with symmetric form

$$\frac{x+5}{3} = \frac{y-1}{7} = \frac{z+10}{-2}.$$

26. Are the two lines with symmetric forms

$$\frac{x-1}{5} = \frac{y+2}{-3} = \frac{z+1}{4}$$

and

$$\frac{x-4}{10} = \frac{y-1}{-5} = \frac{z+5}{8}$$

the same? Why or why not?

27. Show that the two sets of equations

$$\frac{x-2}{3} = \frac{y-1}{7} = \frac{z}{5} \quad \text{and} \quad \frac{x+1}{-6} = \frac{y+6}{-14} = \frac{z+5}{-10}$$

actually represent the same line in $\mathbf{R}^3$.

28. Determine whether the two lines l_1 and l_2 defined by the sets of parametric equations l_1: $x = 2t - 5$, $y = 3t + 2$, $z = 1 - 6t$, and l_2: $x = 1 - 2t$, $y = 11 - 3t$, $z = 6t - 17$ are the same. (Hint: First find two points on l_1 and then see if those points lie on l_2.)

29. Do the parametric equations l_1: $x = 3t + 2$, $y = t - 7$, $z = 5t + 1$, and l_2: $x = 6t - 1$, $y = 2t - 8$, $z = 10t - 3$ describe the same line? Why or why not?

30. Do the parametric equations $x = 3t^3 + 7$, $y = 2 - t^3$, $z = 5t^3 + 1$ determine a line? Why or why not?

31. Do the parametric equations $x = 5t^2 - 1$, $y = 2t^2 + 3$, $z = 1 - t^2$ determine a line? Explain.

32. A bird is flying along the straight-line path $x = 2t + 7$, $y = t - 2$, $z = 1 - 3t$, where t is measured in minutes.

(a) Where is the bird initially (at $t = 0$)? Where is the bird 3 minutes later?

(b) Give a vector that is parallel to the bird's path.

(c) When does the bird reach the point $\left(\frac{34}{3}, \frac{1}{6}, -\frac{11}{2}\right)$?

(d) Does the bird reach $(17, 4, -14)$?

33. Find where the line $x = 3t - 5$, $y = 2 - t$, $z = 6t$ intersects the plane $x + 3y - z = 19$.

34. Where does the line $x = 1 - 4t$, $y = t - 3/2$, $z = 2t + 1$ intersect the plane $5x - 2y + z = 1$?

35. Find the points of intersection of the line $x = 2t - 3$, $y = 3t + 2$, $z = 5 - t$ with each of the coordinate planes $x = 0$, $y = 0$, and $z = 0$.

36. Show that the line $x = 5 - t$, $y = 2t - 7$, $z = t - 3$ is contained in the plane having equation $2x - y + 4z = 5$.

37. Does the line $x = 5 - t$, $y = 2t - 3$, $z = 7t + 1$ intersect the plane $x - 3y + z = 1$? Why?

38. Find where the line having symmetric form

$$\frac{x-3}{6} = \frac{y+2}{3} = \frac{z}{5}$$

intersects the plane with equation $2x - 5y + 3z + 8 = 0$.

39. Show that the line with symmetric form

$$\frac{x-3}{-2} = y - 5 = \frac{z+2}{3}$$

lies entirely in the plane $3x + 3y + z = 22$.

40. Does the line with symmetric form

$$\frac{x+4}{3} = \frac{y-2}{-1} = \frac{z-1}{-9}$$

intersect the plane $2x - 3y + z = 7$?

41. Let a, b, c be nonzero constants. Show that the line with parametric equations $x = at + a$, $y = b$, $z = ct + c$ lies on the surface with equation $x^2/a^2 + y^2/b^2 - z^2/c^2 = 1$.

42. Find the point of intersection of the two lines $l_1: x = 2t + 3$, $y = 3t + 3$, $z = 2t + 1$ and $l_2: x = 15 - 7t$, $y = t - 2$, $z = 3t - 7$.

43. Do the lines $l_1: x = 2t + 1$, $y = -3t$, $z = t - 1$ and $l_2: x = 3t + 1$, $y = t + 5$, $z = 7 - t$ intersect? Explain your answer.

44. (a) Find the distance from the point $(-2, 1, 5)$ to any point on the line $x = 3t - 5$, $y = 1 - t$, $z = 4t + 7$. (Your answer should be in terms of the parameter t.)

(b) Now find the distance between the point $(-2, 1, 5)$ and the line $x = 3t - 5$, $y = 1 - t$, $z = 4t + 7$. (The distance between a point and a line is the distance between the given point and the *closest* point on the line.)

45. (a) Describe the curve given parametrically by

$$\begin{cases} x = 2\cos 3t \\ y = 2\sin 3t \end{cases} \quad 0 \le t < \frac{2\pi}{3}.$$

What happens if we allow t to vary between 0 and 2π?

(b) Describe the curve given parametrically by

$$\begin{cases} x = 5\cos 3t \\ y = 5\sin 3t \end{cases} \quad 0 \le t < \frac{2\pi}{3}.$$

(c) Describe the curve given parametrically by

$$\begin{cases} x = 5\sin 3t \\ y = 5\cos 3t \end{cases} \quad 0 \le t < \frac{2\pi}{3}.$$

(d) Describe the curve given parametrically by

$$\begin{cases} x = 5\cos 3t \\ y = 3\sin 3t \end{cases} \quad 0 \le t < \frac{2\pi}{3}.$$

46. Suppose that a bicycle wheel of radius a rolls along a flat surface without slipping. If a reflector is attached to a spoke of the wheel at a distance b from the center, the resulting curve traced by the reflector is called a **curtate cycloid**. One such cycloid appears in Figure 32, where $a = 3$ and $b = 2$.

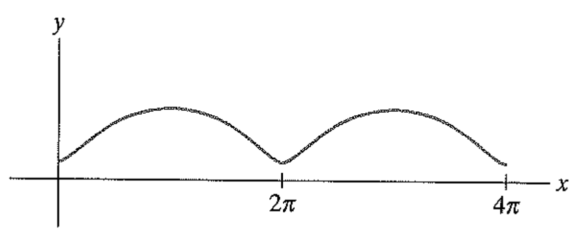

Figure 32 A curtate cycloid.

Using vector methods or otherwise, find a set of parametric equations for the curtate cycloid. Figure 33 should help. (Take a low point of the cycloid to lie