3 Exercises

Compute $\mathbf{a} \cdot \mathbf{b}$, $\|\mathbf{a}\|$, $\|\mathbf{b}\|$ *for the vectors listed in Exercises 1–6.*

1. $\mathbf{a} = (1, 5), \mathbf{b} = (-2, 3)$

2. $\mathbf{a} = (4, -1), \mathbf{b} = \left(\frac{1}{2}, 2\right)$

3. $\mathbf{a} = (-1, 0, 7), \mathbf{b} = (2, 4, -6)$

4. $\mathbf{a} = (2, 1, 0), \mathbf{b} = (1, -2, 3)$

5. $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

6. $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}, \mathbf{b} = -3\mathbf{j} + 2\mathbf{k}$

In Exercises 7–11, find the angle between each of the pairs of vectors.

7. $\mathbf{a} = \sqrt{3}\mathbf{i} + \mathbf{j}, \mathbf{b} = -\sqrt{3}\mathbf{i} + \mathbf{j}$

8. $\mathbf{a} = (-1, 2), \mathbf{b} = (3, 1)$

9. $\mathbf{a} = \mathbf{i} + \mathbf{j}, \mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

10. $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}, \mathbf{b} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

11. $\mathbf{a} = (1, -2, 3), \mathbf{b} = (3, -6, -5)$

In Exercises 12–16, calculate $\text{proj}_{\mathbf{a}}\mathbf{b}$.

12. $\mathbf{a} = \mathbf{i} + \mathbf{j}, \ \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

13. $\mathbf{a} = (\mathbf{i} + \mathbf{j})/\sqrt{2}, \ \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

14. $\mathbf{a} = 5\mathbf{k}, \mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$

15. $\mathbf{a} = -3\mathbf{k}, \mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$

16. $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}, \ \mathbf{b} = 2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$

17. Give a unit vector that points in the same direction as the vector $2\mathbf{i} - \mathbf{j} + \mathbf{k}$.

18. Give a unit vector that points in the direction opposite to the vector $-\mathbf{i} + 2\mathbf{k}$.

19. Give a vector of length 3 that points in the same direction as the vector $\mathbf{i} + \mathbf{j} - \mathbf{k}$.

20. Find three nonparallel vectors that are perpendicular to $\mathbf{i} - \mathbf{j} + \mathbf{k}$.

21. Is it ever the case that $\text{proj}_{\mathbf{a}}\mathbf{b} = \text{proj}_{\mathbf{b}}\mathbf{a}$? If so, under what conditions?

22. Prove properties 2, 3, and 4 of dot products.

23. Prove part 1 of Proposition 3.4.

24. Suppose that a force $\mathbf{F} = \mathbf{i} - 2\mathbf{j}$ is acting on an object moving parallel to the vector $\mathbf{a} = 4\mathbf{i} + \mathbf{j}$. Decompose $\mathbf{F}$ into a sum of vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, where $\mathbf{F}_1$ points along the direction of motion and $\mathbf{F}_2$ is perpendicular to the direction of motion. (Hint: A diagram may help.)

25. In physics, when a constant force acts on an object as the object is displaced, the **work** done by the force is the product of the length of the displacement and the component of the force in the direction of the displacement. Figure 48 depicts an object acted upon by a constant force $\mathbf{F}$, which displaces it from the point P to the point Q. Let θ denote the angle between $\mathbf{F}$ and the direction of displacement.

(a) Show that the work done by $\mathbf{F}$ is determined by the formula $\mathbf{F} \cdot \overrightarrow{PQ}$.

(b) Find the work done by the (constant) force $\mathbf{F} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ in moving a particle from the point $(1, -1, 1)$ to the point $(2, 0, -1)$.

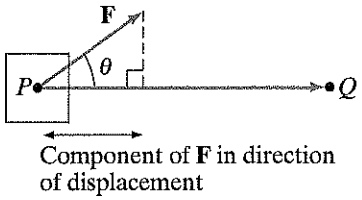

Figure 48 A constant force $\mathbf{F}$ displaces the object from P to Q. (See Exercise 25.)

26. A refrigerator is dragged 12 ft across a smooth floor using a rope and 60 lb of force directed along the rope. How much work is done if the rope makes a 20° angle with the horizontal?

27. How much work is done in pushing a handtruck loaded with 500 lb of bananas 40 ft up a ramp inclined 30° from horizontal?

Let $\mathbf{a}$ *be a nonzero vector in* $\mathbf{R}^3$. *The* ***direction cosines*** *of* $\mathbf{a}$ *are the three numbers* $\cos\alpha$, $\cos\beta$, $\cos\gamma$ *determined by the angles* α, β, γ *between* $\mathbf{a}$ *and, respectively, the positive x-, y-, and z-axes. In Exercises 28 and 29, find the direction cosines of the given vectors.*

28. $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$

29. $\mathbf{a} = 3\mathbf{i} + 4\mathbf{k}$

30. If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, give expressions for the direction cosines of $\mathbf{a}$ in terms of the components of $\mathbf{a}$.

31. Let A, B, and C denote the vertices of a triangle. Let $0 < r < 1$. If P_1 is the point on $\overline{AB}$ located r times the distance from A to B and P_2 is the point on $\overline{AC}$ located r times the distance from A to C, use vectors to show that $\overline{P_1P_2}$ is parallel to $\overline{BC}$ and has r times the length of $\overline{BC}$. (This result generalizes that of Example 6 of this section.)