For a vector $\mathbf{n}$ that is perpendicular to both lines, we may use $\mathbf{n} = \mathbf{a}_1 \times \mathbf{a}_2$, where $\mathbf{a}_1 = (2, 1, 3)$ is a vector parallel to the first line and $\mathbf{a}_2 = (1, -1, 0)$ is parallel to the second. (We may read these vectors from the parametric equations.) Thus,

$$\mathbf{n} = \mathbf{a}_1 \times \mathbf{a}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k},$$

and so,

$$\begin{aligned} \operatorname{proj}_{\mathbf{n}} \overrightarrow{B_1B_2} &= \left(\frac{\mathbf{n} \cdot \overrightarrow{B_1B_2}}{\mathbf{n} \cdot \mathbf{n}} \right) \mathbf{n} = \left(\frac{(-1, -3, 1) \cdot (3, 3, -3)}{(3, 3, -3) \cdot (3, 3, -3)} \right) (3, 3, -3) \\ &= -\tfrac{15}{27}(3, 3, -3) \\ &= -\tfrac{5}{3}(1, 1, -1). \end{aligned}$$

The desired distance is $\|\operatorname{proj}_{\mathbf{n}} \overrightarrow{B_1B_2}\| = \frac{5}{3}\sqrt{3}$. ◆

5 Exercises

1. Calculate an equation for the plane containing the point $(3, -1, 2)$ and perpendicular to $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

2. Find an equation for the plane containing the point $(9, 5, -1)$ and perpendicular to $\mathbf{i} - 2\mathbf{k}$.

3. Find an equation for the plane containing the points $(3, -1, 2)$, $(2, 0, 5)$, and $(1, -2, 4)$.

4. Find an equation for the plane containing the points $(A, 0, 0)$, $(0, B, 0)$, and $(0, 0, C)$. Assume that at least two of A, B, and C are nonzero.

5. Give an equation for the plane that is parallel to the plane $5x - 4y + z = 1$ and that passes through the point $(2, -1, -2)$.

6. Give an equation for the plane parallel to the plane $2x - 3y + z = 5$ that passes through the point $(-1, 1, 2)$.

7. Find an equation for the plane parallel to the plane $x - y + 7z = 10$ that passes through the point $(-2, 0, 1)$.

8. Give an equation for the plane parallel to the plane $2x + 2y + z = 5$ and that contains the line with parametric equations $x = 2 - t$, $y = 2t + 1$, $z = 3 - 2t$.

9. Explain why there is *no* plane parallel to the plane $5x - 3y + 2z = 10$ that contains the line with parametric equations $x = t + 4$, $y = 3t - 2$, $z = 5 - 2t$.

10. Find an equation for the plane that contains the line $x = 2t - 1$, $y = 3t + 4$, $z = 7 - t$ and the point $(2, 5, 0)$.

11. Find an equation for the plane that is perpendicular to the line $x = 3t - 5$, $y = 7 - 2t$, $z = 8 - t$ and that passes through the point $(1, -1, 2)$.

12. Find an equation for the plane that contains the two lines $l_1\colon x = t + 2$, $y = 3t - 5$, $z = 5t + 1$ and $l_2\colon x = 5 - t$, $y = 3t - 10$, $z = 9 - 2t$.

13. Give a set of parametric equations for the line of intersection of the planes $x + 2y - 3z = 5$ and $5x + 5y - z = 1$.

14. Give a set of parametric equations for the line through $(5, 0, 6)$ that is perpendicular to the plane $2x - 3y + 5z = -1$.

15. Find a value for A so that the planes $8x - 6y + 9Az = 6$ and $Ax + y + 2z = 3$ are parallel.

16. Find values for A so that the planes $Ax - y + z = 1$ and $3Ax + Ay - 2z = 5$ are perpendicular.

Give a set of parametric equations for each of the planes described in Exercises 17–22.

17. The plane that passes through the point $(-1, 2, 7)$ and is parallel to the vectors $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{i} - 5\mathbf{k}$

18. The plane that passes through the point $(2, 9, -4)$ and is parallel to the vectors $-8\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ and $3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$

19. The plane that contains the lines l_1: $x = 2t + 5$, $y = -3t - 6$, $z = 4t + 10$ and l_2: $x = 5t - 1$, $y = 10t + 3$, $z = 7t - 2$

20. The plane that passes through the three points $(0, 2, 1)$, $(7, -1, 5)$, and $(-1, 3, 0)$

21. The plane that contains the line $l\colon x = 3t - 5$, $y = 10 - 3t$, $z = 2t + 9$ and the point $(-2, 4, 7)$