Math. 401, Sec. 501   Spring, 2016

Perturbation Theory
Homework # 1, due April 7

Problem 1: Verify the following order relations.

(1) $\epsilon^2 \tanh \epsilon = O(\epsilon^2)$ as $\epsilon \to \infty$.

(2) $\exp(-\epsilon) = o(1)$ as $\epsilon \to \infty$.

(3) $\sqrt{\epsilon(1-\epsilon)} = O(\sqrt{\epsilon})$ as $\epsilon \to 0^+$.

(4) $\dfrac{\sqrt{\epsilon}}{1 - \cos \epsilon} = O(\epsilon^{-3/2})$ as $\epsilon \to 0^+$.

(5) $\epsilon = o(\epsilon^2)$ as $\epsilon \to \infty$.

(6) $\exp(\epsilon) - 1 = O(\epsilon)$ as $\epsilon \to 0$.

(7) $\int_0^\epsilon \exp(-x^2)dx = O(\epsilon)$ as $\epsilon \to 0^+$.

(8) $\exp(\tan \epsilon) = O(1)$ as $\epsilon \to 0$.

(9) $e^{-\epsilon} = o(\epsilon^{-p})$ as $\epsilon \to \infty$, for all $p>0$.

(10) $\ln \epsilon = o(\epsilon^{-p})$ as $\epsilon \to 0^+$, for all $p>0$.

Problem 2: Find only first three terms of the asymptotic power series solution of the following equations.

(1) $x^2 + 2\varepsilon x - 1 = 0$.

(2) $x^3 + \varepsilon x^2 + 1 = 0$.

(3) $x^3 - 4x + 2\varepsilon = 0$.

Problem 3: Find roots of $x^2 - 1 + \varepsilon = 0$ accurate up to $O(\varepsilon^2)$.

1