Math. 401, Sec. 501 Spring, 2016

Homework 2, due Feb 4

1. Construct the full Fourier series of the functions

$$\text{(i)} \quad f(x) = \begin{cases} 1, & -1 \le x \le 0, \\ 0, & 0 < x \le 1; \end{cases}$$

$$\text{(ii)} \quad f(x) = \begin{cases} -1, & -2 \le x \le 0, \\ 2 - x, & 0 < x \le 2; \end{cases}$$

$$\text{(iii)} \quad f(x) = \begin{cases} x^2, & -1 \le x \le 0, \\ 1 + 2x, & 0 < x \le 1; \end{cases}$$

$$\text{(iv)} \quad f(x) = \begin{cases} 0, & -2 \le x \le -1, \\ 1 + x, & -1 < x \le 2. \end{cases}$$

In each case, discuss the convergence of the series on the interval $[-L, L]$ where f is defined and sketch the function to which the series converges pointwise on $[-3L, 3L]$.

2. Construct the Fourier sine series and the Fourier cosine series of the functions

$$\text{(i)} \quad f(x) = \begin{cases} 1, & 0 \le x \le 1, \\ -1, & 1 < x \le 2; \end{cases}$$

$$\text{(ii)} \quad f(x) = \begin{cases} x, & 0 \le x \le 1, \\ -2, & 1 < x \le 2; \end{cases}$$

$$\text{(iii)} \quad f(x) = \begin{cases} 2 + x, & 0 \le x \le 1, \\ 1 - x, & 1 < x \le 2; \end{cases}$$

$$\text{(iv)} \quad f(x) = \begin{cases} 1, & 0 \le x \le 1, \\ 0, & 1 < x \le 2, \\ x - 3, & 2 < x \le 3. \end{cases}$$

In each case, discuss the convergence of the series on the interval $[0, L]$ where f is defined and sketch the functions to which the series converge pointwise on $[-3L, 3L]$.

Answers

1. (i) $f(x) \sim \dfrac{1}{2} + \sum_{n=1}^{\infty} [(-1)^n - 1] \dfrac{1}{n\pi} \sin(n\pi x).$

(ii) $f(x) \sim \sum_{n=1}^{\infty} \left\{ [1 - (-1)^n] \dfrac{2}{n^2\pi^2} \cos \dfrac{n\pi x}{2} + [3 - (-1)^n] \dfrac{1}{n\pi} \sin \dfrac{n\pi x}{2} \right\}.$

(iii) $f(x) \sim \dfrac{7}{6} + \sum_{n=1}^{\infty} \left\{ -\dfrac{2}{n^2\pi^2} \cos(n\pi x) + \left[\dfrac{1 - (-1)^n 2}{n\pi} + 2\dfrac{1 - (-1)^n}{n^3\pi^3} \right] \sin(n\pi x) \right\}.$

(iv) $f(x) \sim \dfrac{9}{8} + \sum_{n=1}^{\infty} \left\{ \dfrac{2}{n^2\pi^2} \left[(-1)^n - \cos \dfrac{n\pi}{2} \right] \cos \dfrac{n\pi x}{2} + \left[(-1)^{n+1} \dfrac{3}{n\pi} + \dfrac{4}{n^2\pi^2} \sin \dfrac{n\pi}{2} \right] \sin \dfrac{n\pi x}{2} \right\}.$

2. (i) $f(x) \sim \sum_{n=1}^{\infty} \dfrac{2}{n\pi} \left[1 + (-1)^n - 2\cos \dfrac{n\pi}{2} \right] \sin \dfrac{n\pi x}{2};$

$f(x) \sim \sum_{n=1}^{\infty} \dfrac{4}{n\pi} \sin \dfrac{n\pi}{2} \cos \dfrac{n\pi x}{2}.$

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(ii) $f(x) \sim \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[-3\cos\frac{n\pi}{2} + \frac{2}{n\pi}\sin\frac{n\pi}{2} + (-1)^n 2\right]\sin\frac{n\pi x}{2};$

$$f(x) \sim -\frac{3}{4} + \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(3\sin\frac{n\pi}{2} + \frac{2}{n\pi}\cos\frac{n\pi}{2} - \frac{2}{n\pi}\right)\cos\frac{n\pi x}{2}.$$

(iii) $f(x) \sim \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[2 + (-1)^n - 3\cos\frac{n\pi}{2} + \frac{4}{n\pi}\sin\frac{n\pi}{2}\right]\sin\frac{n\pi x}{2};$

$$f(x) \sim 1 + \sum_{n=1}^{\infty} \left[\frac{6}{n\pi}\sin\frac{n\pi}{2} + \frac{8}{n^2\pi^2}\cos\frac{n\pi}{2} + (-1)^{n+1}\frac{4}{n^2\pi^2} - \frac{4}{n^2\pi^2}\right]\cos\frac{n\pi x}{2}.$$

(iv) $f(x) \sim \sum_{n=1}^{\infty} \left(\frac{2}{n\pi} - \frac{2}{n\pi}\cos\frac{n\pi}{3} - \frac{2}{n\pi}\cos\frac{2n\pi}{3} - \frac{6}{n^2\pi^2}\sin\frac{2n\pi}{3}\right)\sin\frac{n\pi x}{3};$

$$f(x) \sim \frac{1}{6} + \sum_{n=1}^{\infty} \left[\frac{2}{n\pi}\sin\frac{n\pi}{3} + \frac{2}{n\pi}\sin\frac{2n\pi}{3} + \frac{9}{n^2\pi^2}\cos(n\pi) - \frac{9}{n^2\pi^2}\cos\frac{2n\pi}{3}\right]\cos\frac{n\pi x}{3}.$$

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