Math. 401, Sec. 501 Spring, 2016

Perturbation Theory
Homework # 3, due April 14

1. **Obtain a two-term perturbation expansion for the solutions of the following problems:**

   A. $\frac{dv}{dt} = -1 - \varepsilon v^3, \quad v(0) = 1$

   B. $\frac{dv}{dt} - v = \varepsilon v^2 e^{-t}, \quad v(0) = 1$

   C. $\frac{d^2v}{dt^2} + v = \varepsilon \frac{dv}{dt}, \quad v(0) = 1, \frac{dv}{dt}\Big|_{t=0} = 0.$

   D. $\frac{d^2v}{dt^2} + v = -\varepsilon \left(\frac{dv}{dt}\right)^3, \quad v(0) = 1, \frac{dv}{dt}\Big|_{t=0} = 0.$

2. **Verify the following order relations:**

   (i) $\frac{1-\cos(x^2)}{\sin(x^3)} = \mathrm{O}(\mathrm{x})$ as $x \to 0$

   (ii) $\tan x - x = \mathrm{O}(\mathrm{x}^3)$ as $x \to 0$

   (iii) $\ln(1+\sqrt{x}) - \sqrt{x} = \mathrm{O}(\mathrm{x})$ as $x \to 0$

   (iv) $e^{\sin\varepsilon} - 1 = \mathrm{O}(\varepsilon)$ as $\varepsilon \to 0$

3. **Obtain first three non-zero coefficients in the asymptotic expansion of the following functions using the asymptotic sequence** $\{1, \sin\varepsilon, (\sin\varepsilon)^2, (\sin\varepsilon)^3, \cdots\}$

   (i) $\ln(1+\varepsilon)$

   (ii) e^{ε}

   what would be the coefficients if, instead you use the asymptotic sequence $\{1, \ln(1+\varepsilon), \ln(1+\varepsilon^2) + \cdots\}$. Using both asymptotic series for each of the functions above, find numerical values of $\ln(1.1)$ and $e^{0.1}$.