Math. 401, Sec. 501 — Spring, 2016

Perturbation Theory
Homework 4, due April 26

1. [BUSH (page 84)] Use the Lindstedt–Poincaré technique to show that the two-term uniformly valid expansion of the solution of
$$\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + u = \varepsilon u \left(\frac{\mathrm{d}u}{\mathrm{d}t}\right)^2, \quad u(0) = 1, \quad \frac{\mathrm{d}u}{\mathrm{d}t}(0) = 0,$$
is
$$u = \cos\tau + \frac{\varepsilon}{32}(\cos 3\tau - \cos\tau) + \mathrm{O}(\varepsilon^2)$$
where
$$\tau = \left(1 - \frac{\varepsilon}{8} + \mathrm{O}(\varepsilon^2)\right) t \quad \text{as}\varepsilon \to 0.$$

2. [BUSH (page 97)] Obtain, using the Lindstedt–Poincaré technique, the two-term uniformly valid expansion for the solution of Duffing's equation:
$$\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + u = \varepsilon u^3, \qquad u(0) = 1, \quad \frac{\mathrm{d}u}{\mathrm{d}t}(0) = 0.$$

3. *[Bush, p. 172, (i)]* Find the lowest-order composite solution.
$$\varepsilon y'' - y' + \frac{1}{y} = 0, \quad y(0) = 2, \quad y(1) = 1, \quad 0 \le \varepsilon << 1.$$

4. *[Bush, p. 172, (ii)]* Find the lowest-order composite solution.
$$\varepsilon \frac{d^2y}{dx^2} + \frac{dy}{dx} + e^y = 0, \quad y(0) = 1, \quad y(1) = -\ln 2, \quad 0 \le \varepsilon << 1.$$

5. *[Logan, p. 69, Ex. 3.2(a)]* Find the lowest-order composite solution for each sign of ε.
$$\varepsilon y'' + 2y' + y = 0, \quad y(0) = 0, \quad y(1) = 1, \quad 0 \le |\varepsilon| << 1.$$
Solve this for both signs of ε.