Math. 401, Sec. 501 Spring, 2016

Homework 4, due February 25

1. Use separation of variables to solve the IBVP

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t), \quad 0 < x < 1, t > 0, \\ &u(0,t) = 0, \quad u(1,t) = 0, \quad t > 0, \\ &u(x,0) = f(x), \quad 0 < x < 1, \end{aligned}$$

   when

   (i) $f(x) = \sin(2\pi x) - 3\sin(6\pi x)$;

   (ii) $f(x) = \begin{cases} x & 0 < x \leq \frac{1}{2}, \\ 0, & \frac{1}{2} < x < 1. \end{cases}$

2. Use separation of variables to solve the IBVP

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t), \quad 0 < x < 1, t > 0, \\ &u_x(0,t) = 0, \quad u_x(1,t) = 0, \quad t > 0, \\ &u(x,0) = f(x), \quad 0 < x < 1, \end{aligned}$$

   when

   (i) $f(x) = 3 - 2\cos(4\pi x)$;

   (ii) $f(x) = \begin{cases} 0, & 0 < x \leq \frac{1}{2}, \\ 2x, & \frac{1}{2} < x < 1. \end{cases}$

3. Use separation of variables to solve the IBVP

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t), \quad 0 < x < 1, t > 0, \\ &u(0,t) = 0, \quad u_x(1,t) = 0, \quad t > 0, \\ &u(x,0) = -3. \quad 0 < x < 1, \end{aligned}$$

4. Use separation of variables to solve the IBVP

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t), \quad -1 < x < 1, t > 0, \\ &u(-1,t) = u(1,t), \quad u_x(-1,t) = u_x(1,t), \quad t > 0, \\ &u(x,0) = 2\sin(2\pi x) - \cos(5\pi x), \quad -1 < x < 1. \end{aligned}$$

5. Use separation of variables to solve the IBVP

$$\begin{aligned} &u_{tt}(x,t) = u_{xx}(x,t), \quad 0 < x < 1, t > 0, \\ &u(0,t) = 0, \quad u(1,t) = 0, \quad t > 0, \\ &u(x,0) = 2\sin(3\pi x), \quad u_t(x,0) = 2, \quad 0 < x < 1. \end{aligned}$$

6. Use separation of variables to solve the IBVP

$$\begin{aligned} &u_{tt}(x,t) = u_{xx}(x,t), \quad 0 < x < 1, t > 0, \\ &u(0,t) = 0, \quad u(1,t) = 0, \quad t > 0, \\ &u(x,0) = -3\sin(2\pi x) + 4\sin(7\pi x), \quad u_t(x,0) = \sin(3\pi x), \quad 0 < x < 1. \end{aligned}$$

7. Use separation of variables to solve the IBVP

$$
\begin{aligned}
&u_{xx}(x,y) + u_{yy}(x,y) = 0, \quad 0 < x < 1, 0 < y < 2,\\
&u(x,0) = f_1(x), \quad u(x,2) = f_2(x), \quad 0 < x < 1,\\
&u(0,y) = g_1(y), \quad u(1,y) = g_2(y), \quad 0 < y < 2,
\end{aligned}
$$

when

(i) $f_1(x) \equiv 0,\ f_2(x) = x,\ g_1(y) \equiv 0,\ g_2(y) \equiv 0;$

(ii) $f_1(x) \equiv 0,\ f_2(x) \equiv 0,\ g_1(y) = y,\ g_2(y) \equiv 0;$

(iii) $f_1(x) \equiv 0,\ f_2(x) \equiv 0,\ g_1(y) \equiv 0,\ g_2(y) = y.$