Math. 401, Sec. 501 Spring, 2015

Homework 5, due March 10

1. Use eigenfunction expansion to solve the IBVP

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t) + q(x,t), \quad 0 < x < 1, t > 0,\\ &u(0,t) = 0, \quad u(1,t) = 0, \quad t > 0,\\ &u(x,0) = f(x), \quad 0 < x < 1, \end{aligned}$$

with

(i) $q(x,t) = 2t\sin(2\pi x),\ f(x) = \sin(2\pi x) - 5\sin(4\pi x)$;
(ii) $q(x,t) = e^{-t}\sin(3\pi x) - \sin(5\pi x),\ f(x) = \sin(\pi x) + 2\sin(3\pi x)$;

2. Use eigenfunction expansion to solve the IBVP

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t) + q(x,t), \quad 0 < x < 1, t > 0,\\ &u_x(0,t) = 0, \quad u_x(1,t) = 0, \quad t > 0,\\ &u(x,0) = f(x), \quad 0 < x < 1, \end{aligned}$$

with

(i) $q(x,t) = 2 + \cos(2\pi x),\ f(x) = 2\cos(\pi x) - \cos(2\pi x)$;
(ii) $q(x,t) = (1-x)t,\ f(x) = x$.

3. Use eigenfunction expansion to solve the IBVP

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t) + q(x,t), \quad 0 < x < 1, t > 0,\\ &u(0,t) = 0, \quad u_x(1,t) = 0, \quad t > 0,\\ &u(x,0) = f(x), \quad 0 < x < 1, \end{aligned}$$

with

(i) $q(x,t) = \sin(\frac{3}{2}\pi x) - 2\sin(\frac{5}{2}\pi x),\ f(x) = \sin(\frac{3}{2}\pi x)$;
(ii) $q(x,t) = t\sin(\frac{1}{2}\pi x),\ f(x) = \sin(\frac{1}{2}\pi x) + 2\sin(\frac{5}{2}\pi x)$.

4. Solve the following problem using the method of eigenfuncion expansion.

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t) + q(x,t), \quad 0 < x < 1, t > 0,\\ &u(0,t) = 2t^2 + t, \quad u(1,t) = t^2 - 1, \quad t > 0,\\ &u(x,0) = \sin(2\pi x) - 3\sin(6\pi x), \quad 0 < x < 1,\\ &q(x,t) = (4t+1) - (2t+1)x, \quad 0 < x < 1, t > 0. \end{aligned}$$

5. Solve the following problem using the method of eigenfuncion expansion.

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t), \quad 0 < x < 1, t > 0,\\ &u(0,t) = \frac{t^2}{4}, \quad u(1,t) = 1, \quad t > 0,\\ &u(x,0) = x, \quad 0 < x < 1. \end{aligned}$$