Math. 401, Sec. 501 Spring, 2016

Homework 6, due March 31

1. Use the definition of the Fourier transform to find the Fourier transforms of the following functions.

$$\text{(i)}\ \ f(x) = \begin{cases} x, & -1 < x < 1, \\ 0 & \text{otherwise}; \end{cases} \qquad \text{(ii)}\ \ f(x) = \begin{cases} e^x, & -1 < x < 1, \\ 0 & \text{otherwise}. \end{cases}$$

   Express the result in the form $a + ib$ with a, b real.

2. Show that the Fourier transform of $f(x) = \begin{cases} 1, & |x| \leq a \\ 0, & |x| > a \end{cases}$ is $\tilde{f} = \sqrt{\frac{2}{\pi}} \frac{\sin(\omega a)}{\omega}$.

3. Use a suitable Fourier transformation to solve the following problems.

$$\begin{aligned} &u_t(x,t) = u_{xx}(x,t), \quad -\infty < x < \infty,\ t > 0, \\ &u(x,t), u_x(x,t) \to 0 \ \text{ as } x \to \pm\infty,\ t > 0, \\ &u(x,0) = f(x), \quad -\infty < x < \infty, \end{aligned}$$

   with

   (i) $f(x) = -3e^{-x^2}$.

   (ii) $f(x) = (1 - 2x^2)e^{-4x^2}$.

   (iii) $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$

4. Use a suitable Fourier transformation to solve the following problems.

$$\begin{aligned} &u_{tt}(x,t) = c^2 u_{xx}(x,t), \quad -\infty < x < \infty,\ t > 0, \\ &u(x,t), u_x(x,t) \to 0 \ \text{ as } x \to \pm\infty,\ t > 0, \\ &u(x,0) = f(x), \quad u_t(x,0) = 0, \quad -\infty < x < \infty, \end{aligned}$$

   with

   (i) $c = 2, \quad f(x) = 3e^{-2x^2}$.

   (ii) $c = \frac{1}{3}, \quad f(x) = 1/(x^2 + 9)$.