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THIN FILMS AND FINGERING PROBLEMS IN COMPLEX FLOWS

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ABSTRACT

We consider two problems in complex fluids: (i) thickening of thin films in the Landau-Levich problem [1] of dip coating and in motion of long bubbles in capillary tubes [2]. In both of these problems, thickening of thin films is observed experimentally in the presence of interfacial surfactants which has been confirmed experimentally. Considering small concentration of insoluble surfactants at the interfaces with a small variation of it along the interfaces, we theoretically prove these thickening effects using lubrication theory and boundary layer methods. In the case of Landau-Levich problem of dip coating, we obtain a formula for an upper bound in terms of the Marangoni and Capillary number which is then used to show that the upper bound in the "clean" case (without surfactant) is less than the lower-bound in the "surfactant" case. For the slowly and steadily moving long bubble in a capillary tube, the thickening of the thin film left behind the moving front by the presence of surfactant is based on first obtaining a formula of the film thickness in terms of Marangoni number and concentration of interfacial surfactant. A comparison with Bretherton's "clean" case then shows the thickening effect of surfactant. References [3] and [4] have many details on these problems including many references. We are currently studying extension of these results for the complex fluids, specially polymeric fluids, and also studying the fingering problem in complex fluids in a Hele-Shaw cell.

INTRODUCTION

Interests in fluid flow problems involving thin films arise for many reasons which we use to classify all such problems in two broad categories: (i) problems in which thin films, whether made artificially or naturally, serve some useful purpose and problems are designed with that purpose in mind; and (ii) problems in which thin films arise as a secondary effect when the original problems are actually designed with some other primary purpose in mind. In both of these classes problems, thickness of the thin-film and its dependence on flow parameters are more often a subject of interest. Experimentally and numerically, it has been found that, in general, thickness of the thin-film in most of these problems is proportional to the two-third power to the Capillary number $Ca = U\mu/\gamma$ where U is some problem specific velocity (see below), μ is the fluid viscosity and γ is the surface tension. Moreover, experimental and numerical studies have shown that thickness of the thin-films increases in the presence of interfacial surfactant (see an exhaustive list of references in [3] and [4]). Effect of interfacial surfactant is to decrease the surface tension and hence to increase the thickness of the thin-film if we were to ignore the effect of Marangoni force. Without ignoring this effect, thickening effect of the surfactant has been proved in [3] and [4]. In this paper, we briefly outline the similarity between these two broad categories of problems in terms of mathematical formulations and point out some of the key results that prove the thickening effect of interfacial surfactants.

In all these thin film problems, the flow region extends beyond the thin film. The flow region is usually divided into three regions following boundary layer type of approach: interior region, transition region and exterior region. The exterior region is usually the thin film and the interior region is the region of rapid change in flow features. The flow in the transition region in all these thin-film problems is modeled using lubrication theory which is usually solved numerically matching its solution to solutions of exterior and interior regions. We theoretically analyze these equations in the context of two specific problems without any recourse to numerical computation and prove the thickening effect of interfacial surfactant on the thin-film. The two problems are: Landau-Levich-Derjaguin (LLD for short) dragout coating probelm which falls within the category (i) discussed at the beginning of this section; and the motion of long bubbles in horizontal capillary tubes which falls within category (ii) again discussed at the beginning of this section.

LLD DRAGOUT COATING PROBLEM

Landau-Levich-Derjaguin (LLD) dragout coating probelm is a classical problem involving dip coating flows on an infinite flat substrate which is withdrawn from an infinite liquid bath. Figure 1 below shows the set-up of this problem. Here and below all dimensional variables are denoted with an overbar and dimensionless variables without an overbar. Consider a vertical flat plate being pulled out from a horizontal bath of an incompressible fluid. The $\bar{x}$-axis is downward in the direction of gravity force with $\bar{y}$-axis perpendicular to the plate as shown in Fig. 1. The plate velocity $(\overline{U}, 0)$ is vertically upward and its equation is taken to be $\bar{y} = 0$. Equation of the free surface of the liquid bath far from the plate is $\bar{x} = 0$. The film thickness at finite (negative) $\bar{x}$ on the plate is denoted by $\bar{h}$ and the (constant) film thickness far up on the plate is denoted by $\overline{H_c}$ $(= h_{\text{LLD}})$ for the clean case and $\overline{H_s}$ for the case with surfactant. We denote by $\bar{u}$ the velocity component in the $\bar{x}-$direction, by $\bar{p}$ the pressure, and by $\bar{y} = \bar{h}(\bar{x})$ the free surface of the liquid film for $\bar{x} < 0$. The surfactant concentration on the free surface is denoted by Γ and $\gamma(\Gamma)$ denotes surface tension which depends on Γ. The Γ_∞ and γ_∞ are the corresponding values far up on the plate for the surfactant case, $\gamma_c = \gamma(\Gamma = 0)$ denotes the constant surface tension for the clean case. Note that $\gamma_\infty = \gamma_c$ if $\Gamma_\infty = 0$.

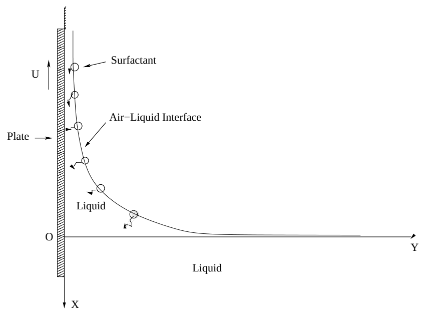

FIGURE 1. Sketch of the drag-out coating problem with coordinates

Introduce dimensionless variables: $(u, v) = (\bar{u}, \bar{v})/U$, $x = \bar{x}/l_c$, $y = \bar{y}/l_c$, $p = \bar{p}l_c/\gamma_c$, $h = \bar{h}/l_c$. As shown in [3], the following equations define the lubrication model for the flow in the transition region:

$$Ca \cdot u_{yy} = p_x - 1, \;\; \& \;\; p_y = 0, \;\; \text{for} \;\; x < 0, \;\; 0 < y < h(x). \quad (1)$$

$$p = -h_{xx}, \;\; \& \;\; Ca \cdot u_y = \gamma_x, \;\; \text{on} \;\; y = h(x). \quad (2)$$

$$u = -1, \;\; \text{on} \;\; y = 0, \quad (3)$$

where $Ca = U\mu/\gamma_\infty$ and γ in (2) is normalized by γ_∞. Below, dimensionless thickness of the thin-film in the clean (surfactant) case is denoted by H_c (H_s). From an analysis of the above equations, it has been shown in Daripa [3] that $H_c < \sqrt{Ca}, \& H_s > \sqrt{Ca}$. Since the strict upper bound for the clean case is also the strict lower bound for the surfactant case, it follows that surfactant thickens the thin film in the Landau-Levich problem.

LONG BUBBLES IN HORIZONTAL CAPILLARY TUBES

We consider here slow steady motion of a long bubble in a horizontal capillary tube of radius r filled with a liquid of viscosity μ. The bubble velocity is denoted by U and the fluid inside the bubble is of negligible viscosity. The flow is considered *axisymmetric*. The Figure 2 shows the set-up. It has been observed experimentally [2] that an almost stagnant thin-film is left behind between bubble and the tube wall as it moves forward. Thus, this thin-film is generated not by design but as a secondary effect unlike the LLD problem discussed above where the primary purpose is to deposit a thin film on the substrate being dragged out of a liquid bath.

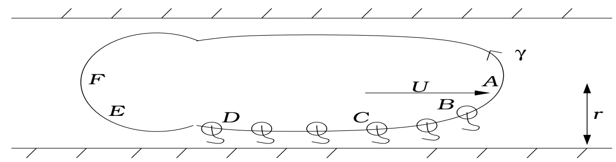

FIGURE 2. The bubble in the capillary tube. The transition region BC is matched with the constant film thickness region CD and with the constant curvature region AB.

Similar to the LLD problem, the thickness of the thin-film here is determined by similar boundary layer approach where bubble interface is decomposed into three regions: the front meniscus *AB*, the transition region *BC* (see Figure 2) and the region of thin-film *CD*. This problem differs from the LLD problem in that the front meniscus here has constant curvature. The equations in the transition region here (see [4]) are similar to the ones given above for LLD-case. Using an analysis of these equations, it is shown in [4] that $b_S > b_C + \frac{3r}{2}M\Gamma(x_B)$ where b_C is the thickness for the clean case, b_S is the thickness when interfacial surfactant is present, M is the Marangoni number and $\Gamma(x_B)$ surfactant concentration at a generic point B in the region BC. This proves the thickening effect of surfactant.

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