Analytical and Numerical Studies of a Singularly Perturbed Boussinesq Equation

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We study the singularly perturbed (sixth-order) Boussinesq equation recently introduced by Daripa and Hua [Appl. Math. Comput. **101** (1999), 159–207]. Motivated by their work, we formally derive this equation from two-dimensional potential flow equations governing the small amplitude long capillary-gravity waves on the surface of shallow water for Bond number very close to but less than 1/3. On the basis of far-field analyses and heuristic arguments, we show that the traveling wave solutions of this equation are weakly non-local solitary waves characterized by small amplitude fast oscillations in the far-field. We review various analytical and numerical methods originally devised to obtain this type of weakly non-local solitary wave solutions of the singularly perturbed (fifth-order) KdV equation. Using these methods, we obtain weakly non-local solitary wave solutions of the singularly perturbed (sixth-order) Boussinesq equation and provide estimates of the amplitude of oscillations which persist in the far-field.

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