Recent Advances on Inverse Airfoil Design

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Over the past three decades significant progress has been made in the analytical and computational theory of inverse design of wings and wing sections. The main driving force behind this rapid progress is its application, particularly to design of planes, ship hulls, turbine and compressor blades, to name just a few. In spite of this rapid progress, design problems that arise in different flow regimes, still remain a challenging one to mathematicians and engineers alike for the very simple reason that these problems are usually overposed and the mathematical formulations of these problems pose difficult theoretical and computational challenges. For example, prescription of arbitrary pressure distribution over the entire, as yet unknown airfoil is known to be overposed for the design of subcritical or shockless transonic airfoil. Removal of the overposedness by a priori modification of the arbitrarily prescribed pressure distribution is impossible due to nonlinearity of the underlying mathematical equations. This in turn poses considerable theoretical and computational challenges for solving the inverse problem in an efficient manner.

Correct formulation of the inverse problem usually requires relaxing the condition of overposedness which is usually a nontrivial task. Modification of the boundary condition to remove the overposedness in a mathematically consistent manner poses theoretical and computational difficulties. There is no unique way to correctly formulating the problem and this is reflected in various different procedures for solving the inverse design problem, depending on the way the overposedness is handled. This often leads to additional complications posing interesting mathematical and computational challenges. Successful circumvention of these challenges has applications to aerospace and other industries.

The need for efficient problem formulation, correct methodologies, fast algorithm and efficient implementation of these into user friendly computer codes has led to several developments such as:

(i) The idea of using potential flow equations in the inverse design has a long tradition. It has been successful in designing subcritical as well as supercritical airfoils. However, there are other viable alternative methods within the same potential flow formulation which are not yet fully exploited.

(ii) The methods of complex characteristics and of free boundary of Garabedain and collaborators are computationally successful but still poses considerable theoretical and computational challenges when pressure is described over the entire airfoil.

(iii) The method of fictitious gas of Sobieczky is quite successful in designing supercritical airfoils.

In spite of these and many other developments in inverse design technology, there still remains considerable room for further development in various aspects of inverse design. We will present at least two new numerical methods and the results of computer codes implementing these methods. Motivated by numerical results we have done some analytical study on the problem of overposedness and have derived some exact theoretical results that satisfactorily answer the question of existence of solutions to the overposed inverse problem. Incorporating this exact result into our computational code further improves the performance and computational complexity of our algorithm.

The numerical algorithms to be presented are inherently parallel and are thus suitable for serial as well as parallel machines. Some of the ideas behind these numerical methods for inverse design motivated development of fast and parallel numerical methods for quasi-conformal mappings and for solving elliptic partial differential equations with possibility of applications to problems in aerodynamics. These methods and ideas will also be discussed.