A Class of Model Equations for Bi-directional Propagation of Capillary-Gravity Waves

R. K. Dash and Prabir Daripa
Department of Mathematics, Texas A&M University,
College Station, TX 77843

A class of model equations that describe the bi-directional propagation of small amplitude long waves on the surface of shallow water is derived from two-dimensional potential flow equations at various order of approximations in two small parameters, namely the amplitude parameter $\alpha = a/h_0$ and wavelength parameter $\beta = (h_0/l)^2$, where a and l are the actual amplitude and wavelength of the surface wave, and h_0 is the height of the undisturbed water surface from the flat bottom topography. These equations are also characterized by the surface tension parameter, namely the Bond number $\tau = \Gamma/\rho g h_0^2$, where Γ is the surface tension coefficient, ρ is the density of water, and g is the acceleration due to gravity.

The traveling solitary wave solutions are explicitly constructed for a class of lower order Boussinesq system. From the Boussinesq equation of higher order, the appropriate equations to model solitary waves are derived under appropriate scaling in two specific cases: (i) $\beta \ll (1/3 - \tau) \leq 1/3$ and (ii) $(1/3 - \tau) = O(\beta)$. The case (i) leads to the classical Boussinesq equation whose fourth-order dispersive term vanishes for $\tau = 1/3$. This emphasizes the significance of the case (ii) that leads to a sixth-order Boussinesq equation, which was originally introduced on a heuristic ground by Daripa and Hua [Appl. Math. Comput. **101**, 159–207, 1999] as a dispersive regularization of the ill-posed fourth-order Boussinesq equation.

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