A Fast Algorithm to Solve the Beltrami Equation with Applications to Quasiconformal Mappings

P. Daripa

Department of Mathematics, Texas A&M University, College Station, TX 77843 Received August 16, 1991

Two algorithms are provided for the fast and accurate computation of the solution of Beltrami equations in the complex plane in the interior of a unit disk. There are two integral operators which are fundamental in the construction of this solution. A fast algorithm to evaluate one of these integrals is given by Daripa (SIAM J. Sci. Stat. Comput., Nov. 1992). An algorithm for fast evaluation of the second integral is provided here. These algorithms are based on representation of the solution in terms of a double integral, some recursive relations in Fourier space, and fast Fourier transforms. The theoretical computational complexity of our algorithm is $O(MN \ln N)$, where M and N are the number of nodes in the circular and radial directions. We show the application of these algorithms in constructing numerical methods for quasiconformal mappings. A numerical method is provided and explored numerically for the construction of quasiconformal mappings using the Beltrami equation. In particular we show the effect of dilatations on the Jacobian of the mappings which controls what is known as the crowding phenomenon in the literature. © 1993 Academic Press, Inc.

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