On a Boundary Control Approach to Domain Embedding Methods

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In this paper, we propose a domain embedding method associated with an optimal boundary control problem, with boundary observations, to solve elliptic problems. We prove that the optimal boundary control problem has an unique solution if the controls are taken in a finite dimensional subspace of the space of the boundary conditions on the auxiliary domain.

Using a controllability theorem due to J.L. Lions, we prove that the solutions of Dirichlet (or Neumann) problems can be approximated within any prescribed error, however small, by solutions of Dirichlet (or Neumann) problems in the auxiliary domain taking an appropriate subspace for such an optimal control problem. We also prove that the results obtained for the interior problems hold for the exterior problems. Some numerical examples are given for both the interior and the exterior Dirichlet problems.