Note on Finite Difference Approximations to Burgers' Equation

H. Aref and P.K. Daripa Division of Engineering, Brown University, Providence, RI 02912

Standard finite difference approximations to Burgers' equation are considered from the point of view of dynamical systems theory. Phase plane analyses for discretizations with a few grid points are presented. These show the existence of initial conditions leading to spurious solutions with unlimited amplitude growth due to nonconservation of kinetic energy by the nondissipative terms in the discretizations. It is shown that such solutions may be found even for arbitrarily fine pointwise resolution, i.e., for arbitrarily many grid points. On the other hand, an energy conserving discretization of the nondissipative terms removes all spurious solutions of this kind. The results obtained seem to complement recent investigations of the steady state problem.

Appeared: SIAM J. Sci. Stat. Comput., 5(4), pp. 856–864, 1984.