A Fast Algorithm to Solve Nonhomogeneous Cauchy-Riemann Equations in the Complex Plane

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An algorithm is provided for the fast and accurate computation of the solution of nonhomogeneous Cauchy-Riemann equations in the complex plane in the interior of a unit disk. The algorithm is based on the representation of the solution in terms of a double integral, some recursive relations in Fourier space, and fast Fourier transforms. The numerical evaluation of the solution at N^2 points on a polar coordinate grid by straightforward summation for the double integral would require $O(N^2)$ floating point operations per point. Evaluation of these integrals has been optimized in this paper giving an asymptotic operation count of $O(\ln N)$ per point on the average. In actual implementation, the algorithm has even better computational complexity, approximately of the order of $O(1)$ per point. The algorithm has the added advantage of working in place, meaning that no additional memory storage is required beyond that of the initial data. The performance of the algorithm has been demonstrated on several prototype problems. The algorithm has applications in many areas, particularly fluid mechanics, solid mechanics, and quasi-conformal mappings.

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