Section 3.1: Limits (PREP WORK)

The first historic problem of Calculus (the tangent line)

Go to the website www.math.tamu.edu/~dmanuel/math151/tangent.gif, which shows the function $f(x) = x^2$ and the tangent line through the point (1, 1). Explain what a tangent line is and why you cannot state exactly the slope of the line based on the given information.

Let $f(x) = x^2$. Find the slope of the line which passes through the following points:

a) $x = 1$ and $x = 2$

b) $x = 1$ and $x = 1.5$

c) $x = 1$ and $x = 1.1$

Section 3.1: Limits

Definition/Idea: $\lim_{x \to a} f(x) = L$ means

$\lim_{x \to a^-} f(x) = L$:

$\lim_{x \to a^+} f(x) = L$:

Examples:

$$\lim_{x \to 4} \frac{x^2 - 5x + 4}{x - 4} \qquad \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \qquad \lim_{x \to 4} \frac{x^2 - 5x - 4}{x - 4}$$

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Laws for Calculating Limits: Suppose $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Then...

$\lim_{x\to a}(f(x) \pm g(x)) =$

$\lim_{x\to a}(c \cdot f(x)) =$

$\lim_{x\to a}(f(x) \cdot g(x)) =$

$\lim_{x\to a} \frac{f(x)}{g(x)} =$ PROVIDED

$\lim_{x\to a}(f(x))^n =$

Two Key Limits:

$\lim_{x\to a} c =$ $\lim_{x\to a} x =$

How to Use these Laws (then see that you don't have to):

Algebraically evaluate $\lim_{x\to 1} x^3 - 3x^2 + 6$

Algebraically evaluate $\lim_{x \to 4} \frac{x^2 - 5x + 4}{x - 4}$.

Algebraically evaluate $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$

If $f(x) = \begin{cases} 1+2x & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ \dfrac{11}{2} - \dfrac{1}{4}x & \text{if } x > 2 \end{cases}$, algebraically evaluate $\lim_{x \to 2} f(x)$

Algebraically evaluate $\lim_{x \to 4} \dfrac{\frac{1}{x} - \frac{1}{4}}{x - 4}$