Exam 2 - An Overview of what you need to know…

Chapter 3 – Linear Programming

- Graph systems of linear inequalities to find the solution (feasible region)
- Determine if a feasible region is bounded or unbounded
- Find the exact value of the corners of the feasible region
- Set up a linear programming problem which includes
    - Defining all the variables
    - Finding the objective function and stating if it is minimized or maximized
    - Stating "Subject to"
    - Listing the constraints, including non-negativity when appropriate
- Know the theorems $f = ax + by$
- Solve linear programming problems using the method of corners, including parametric solutions, word problems and parametric word problems
- After finding the optimal solution, determine if any resources are leftover

Chapter 6 – Sets and Counting

$\{x \mid \sim\sim\}$

- Be able to use roster notation, set notation, and set builder notation
- Find subsets of a set and know the number of subsets and proper subsets
- Know the meaning of the symbols used such as $\varnothing, \in, \notin, \subset, \subseteq, \cup, \cap,$ and ^{c}
- Know how to shade Venn diagrams and express shaded regions in set notation
- DeMorgan's Laws
- ✱ Union rule: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ✱
- Fill in two and three-circle Venn diagrams with numbers when given information
- ✱ $\boxed{\text{Use the multiplication principle to find the number of ways to complete a series of tasks}}$ ✱
- Determine the number of ways to arrange a set of items, some of which are identical $\dfrac{n!}{n_1!\, n_2!\, n_3! \cdots n_k!}$
- Determine the number of ways to choose groups when order does not matter (combinations)

Chapter 7.1 – Experiments, Sample Spaces, and Events

- A sample point is the outcome of an experiment and a sample space is the set of all possible sample points
- Use a tree diagram to find the sample space
- An event is a subset of the sample space
- Mutually exclusive events can't occur at the same time

Part I – Linear Programming

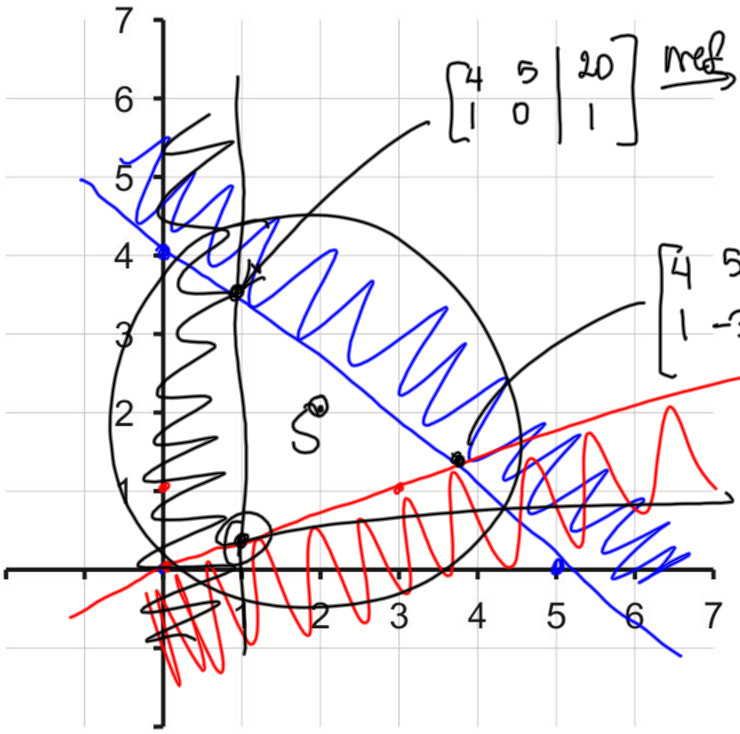
1. A linear programming problem has an objective function $f = 3x - 4y$ on the region

$4x + 5y \leq 20$ (0,4) and (5,0) test (0,0) ⇒ $0 \leq 20$

$x - 3y \leq 0$ (0,0) and (3,1) test (0,1) ⇒ $-3 \leq 0$

$x \geq 1$

What are the maximum and minimum values of f and where are they located?



$\left[\begin{array}{cc|c} 4 & 5 & 20 \\ 1 & 0 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3.2 \end{array}\right] \Rightarrow (1, 3.2)$

$\left[\begin{array}{cc|c} 4 & 5 & 20 \\ 1 & -3 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & \text{cp} \\ 0 & 1 & \text{auc} \end{array}\right]$ math FRAC

$(60/17, 20/17)$

$\left[\begin{array}{cc|c} 1 & -3 & 0 \\ 1 & 0 & 1 \end{array}\right] \xrightarrow{\text{rref}} (1, 1/3)$

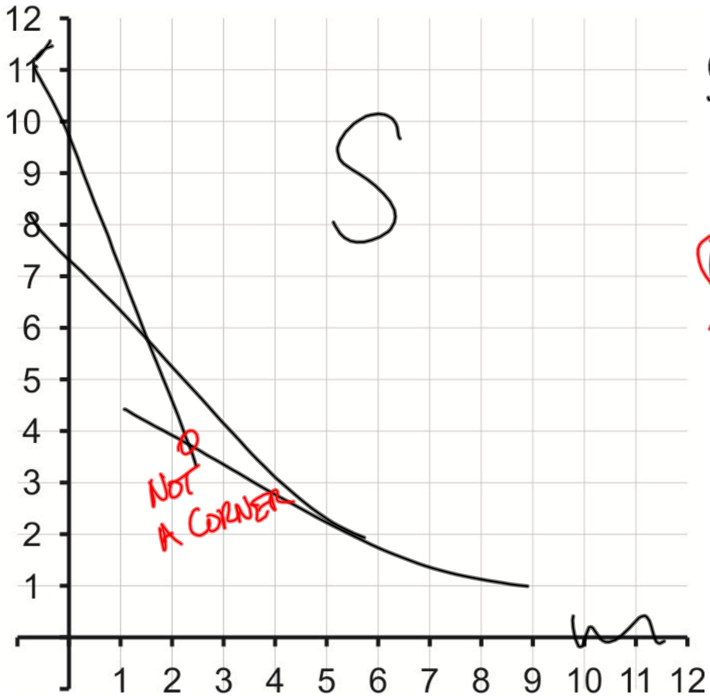
Corners	$f = 3x - 4y$
(1, 3.2)	$3(1) - 4(3.2) = -9.8$
(1, 1/3)	$3(1) - 4(1/3) = 5/3$ (≈1.7)
(60/17, 20/17)	$3(60/17) - 4(20/17) = 100/17$ (≈5.9)

S is bounded, so min value is $f = -9.8$ at (1, 3.2) and the max value is $100/17 = f$ at (60/17, 20/17)

2. A linear programming problem has an objective function $f = 2x + 8y$

on the region
$$\begin{aligned} 5x + 2y &\geq 15 \\ 2x + 3y &\geq 12 \\ x + 4y &\geq 10 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

What are the maximum and minimum values of f and where are they located?



Corners	$f = 2x + 8y$
$(0, 7.5)$	60
$(24/11, 30/11)$	$282/11$ (≈ 25.6)
$(3.6, 1.6)$	20
$(10, 0)$	20

NO MAX. Min $f = 20$ occurs on the line segment $f = 2x + 8y = 20$ for $3.6 \leq x \leq 10$

$2x + 8y = 20 \Rightarrow 8y = -2x + 20 \Rightarrow$ $\boxed{y = -\frac{1}{4}x + 2.5 \text{ with } 3.6 \leq x \leq 10}$

3. Set up the following Linear Programming problem

Farmer Blue has 175 plots available to plant short- and long-stemmed strawberries. Each plot of long-stemmed strawberries will yield 40 baskets of strawberries and each plot of short-stemmed will yield 60 baskets of strawberries. He wants to have at least three times as many baskets of long-stemmed strawberries than he does of short-stemmed strawberries. The long-stemmed will sell for $4.00 per basket and the short-stemmed will sell for $3.00 per basket. How many plots of each type of strawberry should Farmer Blue plant to maximize his revenue?

x = # of plots of SS str

y = " " " LS "

R = revenue in $

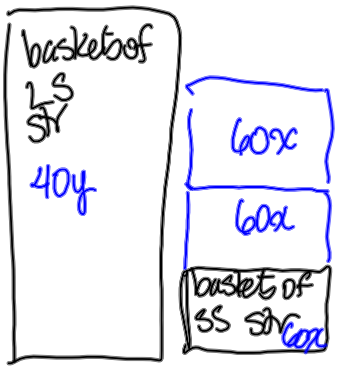
MAX $R = 3(60x) + 4(40y) = 180x + 160y$

SUB TO

$x + y \leq 175$ (plots)

$40y \geq 3(60x)$

$x \geq 0, y \geq 0$



4. A manufacturer makes two types of products: widgets and gadgets. Each widget and gadget needs to be fabricated, polished and wrapped as shown in the table below:

	fabrication minutes	polishing minutes	wrapping minutes	Profit
x=# of widget	9	12	11	$3
y=# of gadgets	9	10	6	$5
available time	288 minutes	338 minutes	275 minutes	

How many of each type of product should be produced to realize a maximum profit? What is the maximum profit? What, if anything is leftover?

x = # of wid
y = # of gad
P = profit in $

Max $P = 3x+5y$
SUB TO
$9x+9y \le 288$ fab min
$12x+10y \le 338$ pol min
$11x+6y \le 275$ wr min
$x \ge 0, y \ge 0$

Vertex	$P=3x+5y$
(0,32)	160 *
(9,23)	142
(19,11)	112
(25,0)	75
(0,0)	0

Max profit is $160 when 0 widgets and 32 gadgets are made

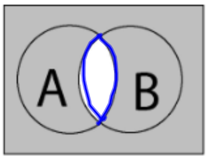
fab: $9(0)+9(32)=288$
0 fab. mins left

$12(0)+10(32)=320$
$338-320=18$ pol. min left

$11(0)+6(32)=192$
$275-192=83$ wrap min left

Part II – Sets

1. Express the shaded regions in set notation:



$(A \cap B)^c$ or $A^c \cup B^c$

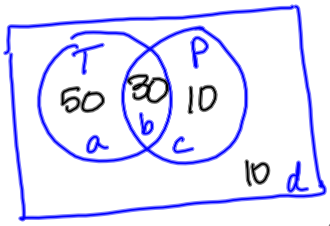
2. A class of math students can be grouped in the following sets:

$A=\{x|x \text{ is a woman}\}$ $\qquad B=\{x|x \text{ has taken Economics}\}$

Find the set of women who have not taken Economics in set builder notation

$\{x| x \in A \text{ and } x \notin B\}$

3. A store has sold 100 microwaves. 80 of the microwaves have turntables and 40 of them have ~~programs~~. If 90 of them have programs or turntables, how many have only turntables?



$100 = a + b + c + d$

$80 = a + b$

$40 = b + c$

$90 = a + b + c$

50

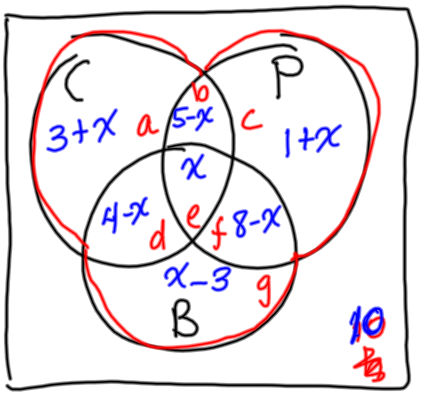
$n(T \cup P) = n(T) + n(P) - n(T \cap P)$

$90 = 80 + 40 - n(T \cap P)$

$\rightarrow n(T \cap P) = 30$

4. A survey of 31 students is done at a school cafeteria. Use the information given to fill in a Venn diagram:

- 12 like cantaloupe $=a+b+d+\cancel{e}$
- 14 like pumpkin $=b+c+\cancel{e}+f$
- 9 like bananas $=d+\cancel{e}+f+g$
- 5 like cantaloupe and pumpkin $b+\cancel{e}$
- 4 like cantaloupe and bananas $d+\cancel{e}$
- 8 like pumpkin and bananas $\cancel{e}+f$
- 10 liked none of these items h



$$21 = 3+x + 5-x + 1+x + 4-x + x + 8-x + x-3$$

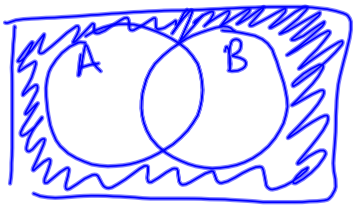
$$21 = x+18 \Rightarrow x=3$$

5. Shade the region corresponding to

(a) $\{x \mid \underbrace{x \notin A}\ \text{or}\ x \in B\}$

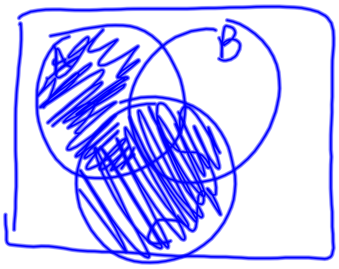
JOIN

(b) $(A \cup B)^c$

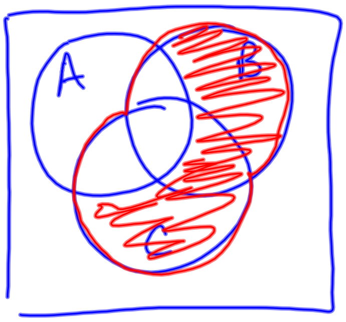


$A^c \cap B^c$

(c) $(A \cap B^c) \cup C$

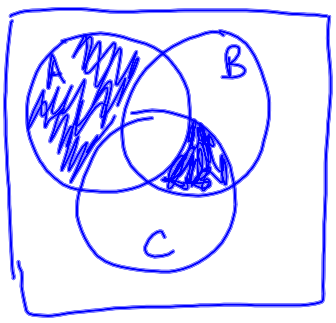


(d) $(B \cup C) \cap A^c$



(e) $\left(A \cap B^C \cap C^C\right) \cup \left(A^C \cap B \cap C\right)$

join



6. Given the following sets, decide if each statement is true or false. Note that U is the universal set.

$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ $A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$ $C = \{3, 5, 7\}$

(a) B and C are disjoint True b/c $B \cap C = \emptyset$

$1 \in A$ is true

$\{1\} \subseteq A$ $\{1\} \subset A$

(b) $1 \subseteq A$ False

$B \subseteq B$ true

(c) $B \subset B$ False

(d) $\{3,5\} \in C$ false $\{3,5\} \subseteq C$ or $\{3,5\} \subset C$

or $3,5 \in C$

$A = \{1,2,3\}$, $\{B\} = \{2,4,6\}$

(e) $A \cap B = 2$ False $A \cap B = \{2\}$

(f) $A \cup C = \{1,2,3,\cancel{4},5,7\}$

(g) A has 8 proper subsets false

$2^3 - 1 = 7$ proper subsets

Part III – Counting

1. There are three letters in an airport's abbreviation.

(a) How many different airport call letters are possible?

$\underset{L}{\underline{26}} \cdot \underset{L}{\underline{26}} \cdot \underset{L}{\underline{26}} = 17,576 \Rightarrow$ AAA AAB AAC ....

.... BBB BBC ...

.. CCC

etc

ZZZ

(b) How many are possible if no repeats are allowed?

$\underset{L}{\underline{26}} \cdot \underset{L}{\underline{25}} \cdot \underset{L}{\underline{24}} = 15,600$ $(P(26,3))$

$26 \cdot 26 \cdot 25$ AAB But not ABA or BAA

(c) How many if three of the same letter is not allowed?

$\underset{\text{no restr}}{\underline{17576}} - \underset{\text{not allowed}}{\underline{26}} = 17,550$

(d) How many are possible if no repeats are allowed and a vowel must be in the middle?

$\underline{25} \cdot \underset{V}{\underline{5}} \cdot \underline{24} = 3000$

2. A pizza place has 12 different toppings available for pizza. How many different 2 item pizzas are possible?

$$\underbrace{12}_{\text{\# of ways to get a pizza w/ 2 of the same topping}} + \underbrace{C(12,2)}_{\text{\# of ways to get a pizza w/ 2 different toppings}} = 12 + 66 = 78$$

or

3. How many different "words" can be made from the letters in *HALLOWEEN*?

$$\frac{9!}{\underbrace{1!}_{H}\,\underbrace{1!}_{A}\,\underbrace{2!}_{L}\,\underbrace{1!}_{O}\,\underbrace{1!}_{W}\,\underbrace{2!}_{E}\,\underbrace{1!}_{N}} = 9!/(2!\cdot 2!) = 90,720$$

4. You are dealt 2 cards. How many ways can you be dealt a blackjack? (that is, a sum of 21 where an ace is worth 11 and a 10 and face cards are worth 10).

$\underset{\text{ace}}{\underline{4}} \cdot \underset{\text{10pt}}{\underline{16}} = 64$

$\left[C(52,2) = 1326 \right]$

5. From a class of 18 students a committee of 5 is chosen. One person on the committee is the chair and the others are the members. How many different committees can be chosen?

$$\underbrace{18}_{ch} \cdot \underbrace{C(17,4)}_{rest} = 42840$$

$$\underbrace{C(18,5)}_{\text{committee}} \cdot \underbrace{5}_{\text{chair}} = 42,840$$

6. You have 4 different yellow books and 4 different green books. How many ways can the books be arranged on the shelf if the colors must alternate?

$\underset{1^{st}}{\underline{8}} \cdot \underset{2^{nd}}{\underline{4}} \cdot \underline{3} \cdot \underline{3} \cdot \underline{2} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1}$

$\cancel{Y_1}\ \cancel{Y_2}\ Y_3\ Y_4\ \cancel{G_1}\ \cancel{G_2}\ G_3\ G_4$

7. You have a bag of jelly beans. There are 4 blue, 5 green and 2 pink jelly beans. A sample of 3 is chosen. How many ways to have exactly one blue or exactly two green?

exactly 1 Blue

BPP, BPG, BGG

GGP, GGB

ex 2 green

E is one Blue: $n(E) = \underbrace{C(4,1)}_{1B} \cdot \underbrace{C(7,2)}_{2B^c} = 84$ and

F is 2G: $n(F) = \underbrace{C(5,2)}_{2G} \cdot \underbrace{C(6,1)}_{1G^c} = 60$ and

$n(E \cap F) = \underbrace{C(5,2)}_{2G} \cdot \underbrace{C(4,1)}_{1B} = 40$ and

$n(E \cup F) = n(E) + n(F) - n(E \cap F)$

$= 84 + 60 - 40 = 104$

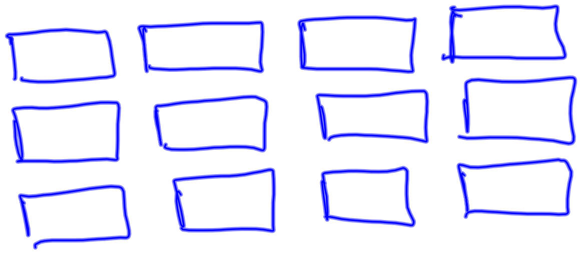
8. How many ways can a hand of 4 cards be dealt if exactly 3 of them are queens?

$\underbrace{C(4,3)}_{3Q} \cdot \underbrace{C(48,1)}_{1Q^c} = 192$

and

. You have advertisement to layout for a full page newspaper ad. The ad features 3 different departments and each department has 4 items for sale. If items from the same department must appear all on the same row, how many different ads are possible?

Depts A, B, and C

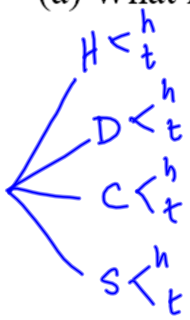


$$\underbrace{(3\cdot 2\cdot 1)}_{\text{arr the rows}} \cdot \underbrace{(4\cdot 3\cdot 2\cdot 1)}_{\text{arr Dept A}} \cdot \underbrace{4!}_{\text{arr Dept B}} \cdot \underbrace{4!}_{C} = 82{,}944$$

Part IV – Language of Probability

1. An experiment consists of choosing a card from a standard deck of cards and noting the suit and then flipping a coin.

(a) What is the sample space for this experiment?



$S = \{Hh, Ht, Dh, Dt, Ch, Ct, Sh, St\}$

$n(S) = 8 = \underset{\text{card}}{4} \cdot \underset{\text{coin}}{2}$

(b) How many events are possible in this experiment?

$2^8 = 256$

(c) Are the events E, a heart is picked and F, a head is tossed mutually exclusive events?

Not M/E, an outcome is Hh