

WEEK 7 REVIEW: SETS and the COUNTING*Example*

Let $U = \{x \mid x \text{ is a positive integer less than } 8\}$,
 $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$, and $C = \{5, 6, 7\}$

a. Write U in roster notation

b. $A^c =$

c. $A \cap B =$

d. $A \cup B =$

e. List all the subsets of C

Determine if the statements below are true or false

f. A and C are disjoint sets

g. $\emptyset \in A$

h. $\{5, 6, 7\} \subset C$

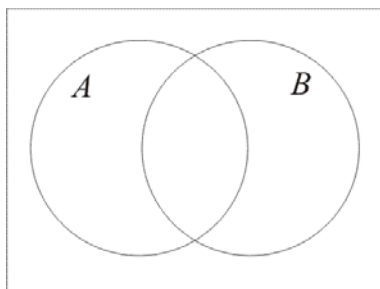
Example: Use Venn diagrams to indicate

- a. $A \subset U, B \subset U, A \subset B^c$ b. $A \subset U, B \subset U, C \subset U, C \subset A \cap B$

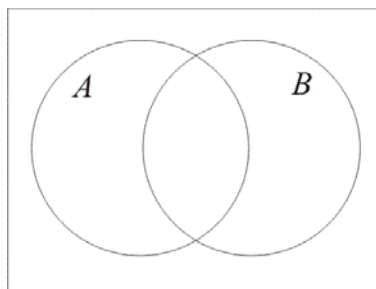
Example

Shade the indicated regions on the Venn diagram

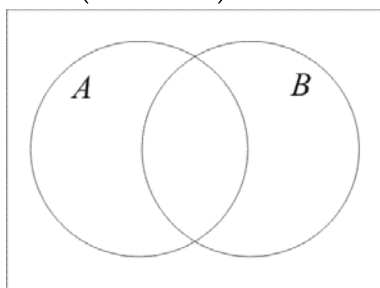
- a. $A \cap B^c$



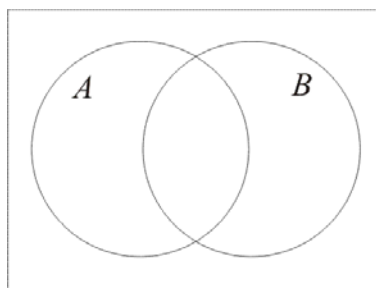
- b. $A \cup B^c$



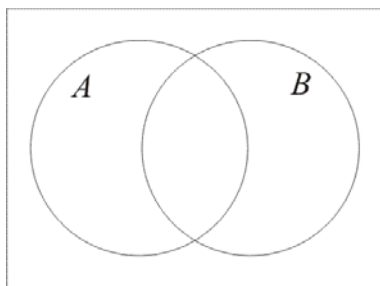
- c. $(A \cap B)^c$



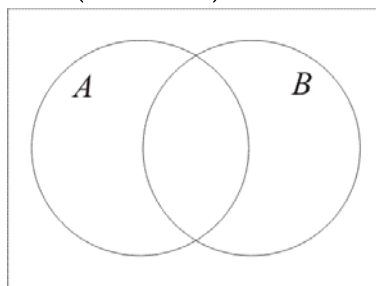
- d. $A^c \cup B^c$



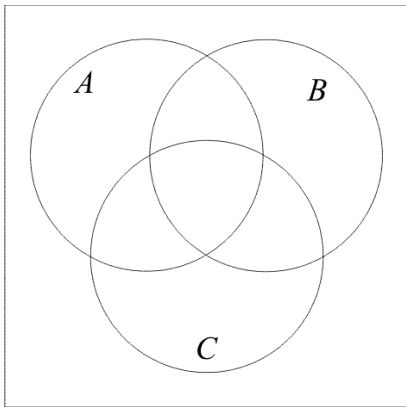
- e. $A^c \cap B^c$



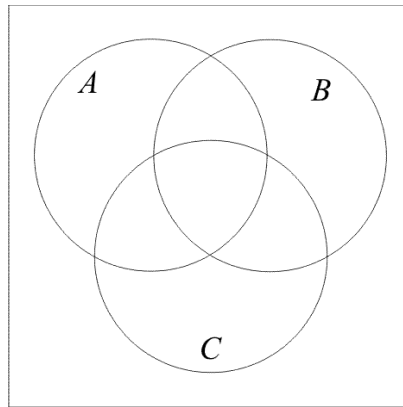
- f. $(A \cup B)^c$



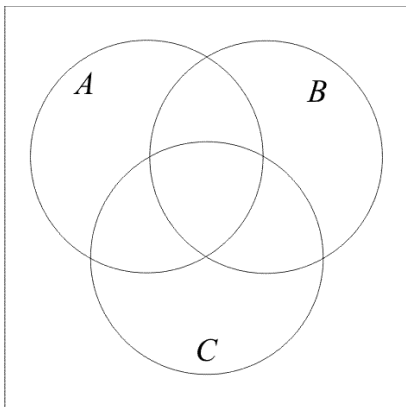
g. $A^c \cap B \cap C$



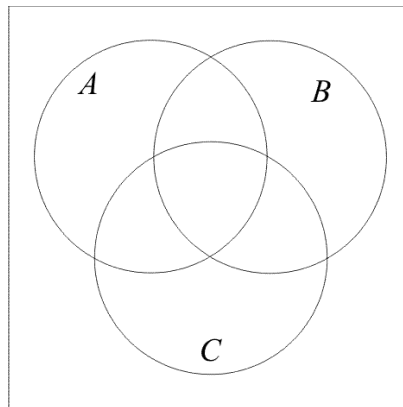
h. $(A \cup B) \cap C^c$



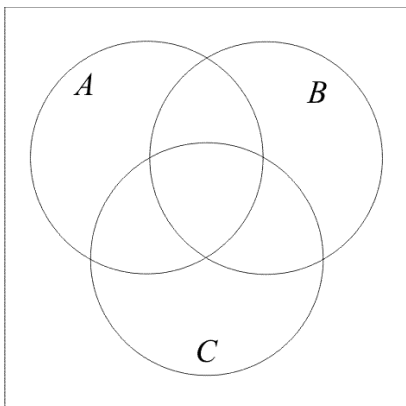
i. $(A^c \cap B)^c \cup C$



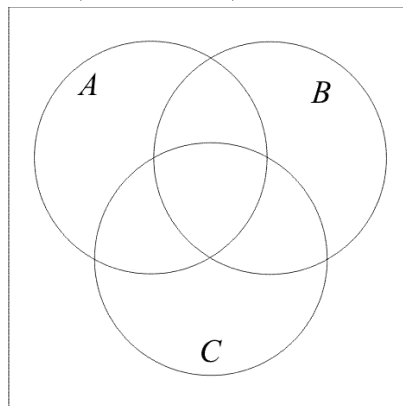
j. $A^c \cup B \cup C$



k. $(A \cup B)^c \cap C$



l. $(A^c \cap B)^c \cap C$



Example

Let U be the set of all staff at Texas A&M University and let

$$A = \{x|x \text{ owns an automobile}\}$$

$$H = \{x|x \text{ owns a house}\}$$

$$P = \{x|x \text{ owns a piano}\}$$

Describe the following sets in words

a. A^c

b. $A \cap H^c$

c. $A^c \cup P^c$

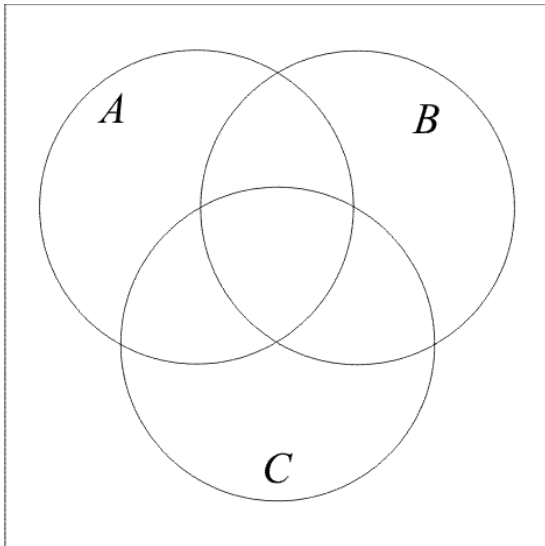
d. $A^c \cap H^c \cap P^c$

Example

If $n(A) = 100$, $n(A \cap B) = 20$, and $n(A \cup B) = 150$, what is $n(B)$?

Example

Given $n(U) = 100$, $n(A) = 40$, $n(B) = 37$, $n(C) = 35$,
 $n(A \cap B) = 25$, $n(A \cap C) = 22$, $n(B \cap C) = 24$, and
 $n(A \cap B \cap C) = 10$, find $n(A^c \cap B \cap C)$.

*Example*

In a survey of 120 adults, 55 said they had an egg for breakfast that morning, 40 said they had juice for breakfast, and 70 said they had an egg or juice for breakfast. How many had an egg but no juice for breakfast? How many had neither an egg nor juice for breakfast?

Example

Determine how many pizzas were sold if

- 3 pizzas had mushrooms, pepperoni, and sausage
- 7 pizzas had pepperoni and sausage
- 6 pizzas had mushrooms and sausage but not pepperoni
- 15 pizzas had two or more of these toppings
- 11 pizzas had mushrooms
- 8 pizzas had only pepperoni
- 24 pizzas had sausage or pepperoni
- 17 pizzas did not have sausage

Example

Six hundred people were surveyed and it was found that during the past year,

- 330 did not travel by bus,
- 100 traveled by plane but not by train,
- 150 traveled by train but not by plane,
- 120 traveled by bus but not by train or plane,
- 100 traveled by both bus and plane,
- 40 traveled by all three, and
- 220 traveled by plane.

How many did not travel by any of these three modes of transportation?

Example

At a pasta diner there is a choice of 4 different pastas and 3 different sauces. How many dinners can be made?

Example: How many different 4-digit access codes can be made if

- a. there are no restrictions?
- b. there are no repeats?
- c. the first digit cannot be a 0 or a 1 and no repeats are allowed?
- d. four of the same digit is not allowed?

Example

You take a multiple choice test with 3 questions and each question has 5 possible answers. How many ways can the test be answered?

Example

Matthew and Jennifer go to the movies with four of their friends. How many ways can these six children be seated if

- a. there are no restrictions?
- b. Matthew and Jennifer are seated next to each other?
- c. Matthew and Jennifer are not next to each other?

Example

Four couples are going to the movie together. How many ways can these eight people be seated if couples sit together?

Combinations

Example: How many ways can we choose a *group* of 4 students from a class of 10 students?

This is the number of *combinations* of 10 items taken 4 at a time.

$$C(n, r) = \frac{n!}{(n-r)!r!}$$

Example: A lottery has 54 numbers and 6 are chosen to be winning numbers. If the order the numbers are picked doesn't matter, how many ways can these numbers be chosen?

How many ways to choose no winning numbers?

How many ways to choose at least 3 winning numbers?

Example: From 10 names on a ballot, how many ways can 4 be elected to a committee if each one person is the chair and another is the vice-chair?

Example: How many ways can 10 books be arranged on a shelf if there are 3 identical biology books, 6 identical physics books and a math book?

Example: You have a group of 13 different books. Three are math books, four are chemistry and six are history books. How many different arrangements are possible if books of the same type are kept together?

Example: How many ways can 6 girls and 3 boys be arranged in a row if boys cannot be next to each other?