CHAPTER 17 – INFORMATION SCIENCE

Binary and decimal numbers – a short review:

For decimal numbers we have 10 digits available (0, 1, 2, 3, … 9)

$4731 = 4000 + 700 + 30 + 1 = 4\times10^3 + 7\times10^2 + 3\times10^1 + 1\times10^0$

$\underset{10000's}{2}\ \underset{1000's}{5},\ \underset{100's}{9}\ \underset{10's}{7}\ \underset{1's}{8}$

For binary numbers we have 2 digits available, 0 and 1.

64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Express the following binary numbers as decimal numbers:

10101 = $\underset{16}{1}\ \underset{8}{0}\ \underset{4}{1}\ \underset{2}{0}\ \underset{1}{1} \Rightarrow 16 + 4 + 1 = 21$

11100010 = $\underset{128}{1}\ \underset{64}{1}\ \underset{32}{1}\ \underset{16}{0}\ \underset{8}{0}\ \underset{4}{0}\ \underset{2}{1}\ \underset{1}{0} \Rightarrow 128 + 64 + 32 + 2 = 226$

Express the following decimal numbers as binary numbers:

55 = $\underset{32}{1}\ \underset{16}{1}\ \underset{8}{0}\ \underset{4}{1}\ \underset{2}{1}\ \underset{1}{1}$ $\quad 55-32=23,\ 23-16=7$, $7-4=3$

88 = $\underset{64}{1}\ \underset{32}{0}\ \underset{16}{1}\ \underset{8}{1}\ \underset{4}{0}\ \underset{2}{0}\ \underset{1}{0}$ $\quad 88-64=24,\ 24-16=8$

$9 = \underset{8}{1}\ \underset{4}{0}\ \underset{2}{0}\ \underset{1}{1}$

An orbiting satellite can follow 16 different directions that are labeled 0 to 15 in binary (0000 to 1111). Each message is sent as the command along with 3 check digits. The check digits are arranged so that certain sums have even parity. These are called ***parity-check sums*** where the parity of a number refers to whether a number is even or odd. Even numbers have ***even parity*** and odd numbers have ***odd parity***.

For our satellite, the following sums must be even (0 mod 2)
$a_1+a_2+a_3+c_1$, $a_1+a_3+a_4+c_2$, and $a_2+a_3+a_4+c_3$

$a_1\,a_2\,a_3\,a_4$
1001 is 9 in binary

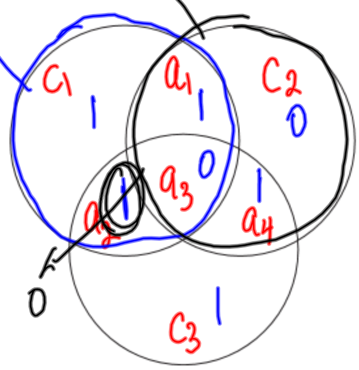
What are the check digits for command 9?

$1+0+0+c_1$ must be even, so $c_1=1$
$1+0+1+c_2$ must be even, so $c_2=0$
$0+0+1+c_3$ must be even, so $c_3=1$

$c_1c_2c_3 = 101$

Codeword: 1001101

For this type of parity-check sum, we can use a Venn diagram to help find the check digits or find errors.



message is 1001

0

Fix the error in the code 1~~1~~01101if it is known only one digit has an error.

A set of words composed of 0's and 1's that has a message and parity check sums appended to the message is called a ***binary linear code***. The resulting strings are called ***code words***.

+

The process of determining the message you were sent is called ***decoding***. If you are sent a message x and receive the message as y, how can it be decoded?

The ***distance between two strings*** of equal length is the number of positions in which the strings differ.

(a) 10101 and 11101

11101

dist is 1

(b) 111111 and 000000

000000

dist is 6

The ***nearest neighbor decoding method*** decodes a message as the code word that agrees with the message in the most positions provided there is only one such message.

How good a code is at detecting and correcting errors is determined by the weight of the code. The ***weight of a binary code*** is the minimum number of 1's that occur among all non-zero code words of that code.

Consider a code of weight t,

- The code can detect $t-1$ or fewer errors.
- If t is odd, the code will correct $\frac{t-1}{2}$ or fewer errors.
- If t is even, the code will correct any $\frac{t-2}{2}$ or fewer errors.

4 4 7

Consider the code C = {0000000, 0001111, 1111000, 1111111}

A B C D

(a) What is the weight of the code? $t = 4$

(b) How many errors can this code detect? $4-1 = 3$ errors

(c) How many errors can this code correct? $\frac{4-2}{2} = 1$ error

(d) Decode the message received as 0001101. B

should be a 1

A ***compression algorithm*** converts data from an easy-to-use format to one that is more compact. jpg photo files use data compression as do most video and audio files.

Delta function encoding uses the differences in one value to the next to encode the data.

The data below is the closing price of the Dow Jones on Oct. 1, 2012 – Oct 5, 2012. Compress the data using delta function encoding and determine how much the data is compressed.

13610 13575 13495 13482 13515 → $5 \times 5 = 25$ char

13610 −35 −80 −13 33 → 16 char

compressed by $25 - 16 = 9$ char

$\frac{1}{4\text{'s}}$ $\frac{0}{2\text{'s}}$ $\frac{1}{1\text{'s}}$

Binary codes can also be compressed by assigning short codes to characters that occur frequently and longer codes to characters that occur rarely.

We have 5 symbols, π, μ, σ, δ, and α. If we give all the symbols a code of the same length, we would need 3 binary digits (000 to 101). So a string of 6 symbols would be 6 x 3 = 18 characters long. Can we devise a different binary code if we knew how often each character occurred?

Use ***Huffman coding*** is a way to assign shorter code words to those characters that occur more often.

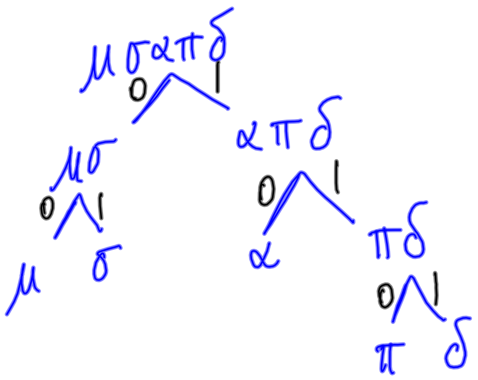
π	μ	σ	δ	α
0.16	0.19	0.23	0.17	0.25

π δ μ σ α
$\underbrace{.16\ \ .17}_{.33}$ .19 .23 .25

μ σ α πδ
$\underbrace{.19\ \ .23}_{.42}$ .25 .33

α πδ μσ
$\underbrace{.25\ \ .33}_{.58}$ .42

μσ απδ
$\underbrace{.42\ \ .58}_{μσαπδ}$



μ	00
σ	01
α	10
π	110
δ	111

DECODE

10|00|01|00|110|00|111|00
α | μ | σ | μ | π | μ | δ | μ

The process of disguising data is called encryption. Cryptology is the study of making and breaking secret codes.

A ***Caesar cipher*** shifts the letters of the alphabet by fixed amount.

EXAMPLE
Create a Caesar cipher that shifts the alphabet by 10 letters and use it to encrypt the message THANKS.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J

DRKXUC

The message below was created with a Caesar cipher with a shift of 14. What is the original message?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N

HSZSDVCBS
TELEPHONE

A ***decimation cipher*** multiplies the position of each letter by a fixed number k (called the ***key***) and then uses modular arithmetic. To use a decimation cipher,

1. Assign the letters A – Z to the numbers 0 – 25.
2. Choose a value for the key, k, that is an odd integer from 3 to 25 but not 13 (why not?)
3. Multiply the value of each letter (i) by the key (k) and find the remainder when divided by 26.
4. To decrypt a message, the encrypted value x needs to be multiplied by the decryption letter j and then the remainder mod 26 is the original letter.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Use a decimation cipher with a key of 11 to encrypt THANKS

	T	H	A	N	K	S
position	19	7	0	13	10	18
x11	209	77	0	143	110	198
mod 26	1	25	0	13	6	16
Letter	B	Z	A	N	G	Q

The message below was encrypted with a key of 21. The decryption key is 5. Decode the message.

	Q	R	G	S	M	O	J	T	K
position	16	17	6	18	12	14	9	19	10
x 5	80	85	30	90	60	70	45	95	50
mod 26	2	7	4	12	8	18	19	17	24
letter	C	H	E	M	I	S	T	R	Y

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

A ***Vigenère cipher*** uses a ***key word*** to encode the characters.

Use a ***Vigenère cipher*** with a key word of MINT to encode the message

position	13	4	22	15	17	8	13	19	4	17
	N	E	W	P	R	I	N	T	E	R
	12	8	13	19	12	8	13	19	12	8
+	M	I	N	T	M	I	N	T	M	I
	25	12	35	34	29	16	26	38	16	25
mod 26	25	12	9	8	3	16	0	12	16	25
	Z	M	J	I	D	Q	A	M	Q	Z

A ***Vigenère cipher*** with a key word of LEX was used to encode the message below. Decode it.

	D	Y	M	P	V	J	L	R
	3	24	12	15	21	9	11	17
−	11	4	23	11	4	23	11	4
	L	E	X	L	E	X	L	E
	−8	20	−11	4	17	−14	0	13
mod 26	18	20	15	4	17	12	0	13
	S	U	P	E	R	M	A	N

To increase security, codes can be added together. Find 10110 + 00111 using binary addition. In "*binary addition*," if the sum is even, enter a 0.

$$\begin{array}{r} 10110 \\ 00111 \\ \hline 10001 \end{array}$$

SAMPLE EXAM QUESTIONS FROM CHAPTER 17

1. Convert the binary number 11001 to a decimal number.

16 8 4 2 1

(A) 3 (B) 25

(C) 6 (D) 31

16 + 8 + 1

2. What is the distance between received words 1100101 and 1010111?

1010111

(A) 1 (B) 2 (C) 3 (D) 4

(E) more than 4

3. Add the binary sequences 1100101 and 1110001. How many 1s digits are in the sum?

1100101

0010100

(A) 1 (B) 2 (C) 3 (D) 4

(E) more than 4

4. ~~Use delta encoding to compress the data~~

1834 1831 1831 1825 1850 ⇒ 1834 -3 0 -6 25

By how many characters is the data compressed?

11 char

(A) 9 (B) 10

(C) 11 (D) 13

4x5=20

5. Use the nearest-neighbor Venn diagram method to decode the received word 1101101.

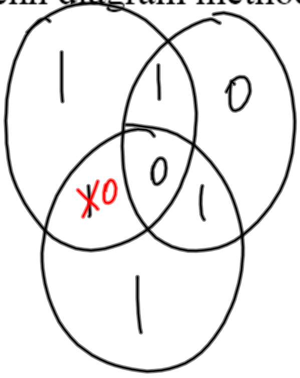
(A) 1001

(B) 0100

(C) 1101

(D) 1011

(E) None of these



Questions 6 and 7 use the code {1100, 1010, 1001, 0110, 0101, 0011}.

2 2 2 2 2 2

6. What is the weight of this code?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

t=2

detect $t-1 = 2-1 = 1$ error

correct $\frac{t-2}{2} = \frac{2-2}{2} = 0$ errors

7. Which one of the following is a true statement about this code?

(A) This code can detect and correct two errors

(B) This code can detect two errors and correct 1 error

(C) This code can detect and correct one error.

(D) This code can detect one error and correct 0 errors

(E) None of these

Question 8 (6 points)

Given binary codes A → 0, C → 10, I → 110, S → 1110, B →11110.

(a) Encode the message CASSI

100111011100110

(b) Decode the message 11110|0|1110|110|10|1110

B A S I C S

Question 9 (5 points) What is the code word for the message 110, if the code word is the message appended with three check digits found using the parity-check sums $\underbrace{a_1 + a_2 + a_3}_{c_1}$, $\underbrace{a_1 + a_3}_{c_2}$ and $\underbrace{a_2 + a_3}_{c_3}$?

$a_1 a_2 a_3$

1+1+0 + c_1 is even so $c_1 = 0$

1+0+c_2 is even, so $c_2 = 1$

1+0+c_3 is even, so c_3 is 1

110011

Question 10 (7 points)

Use a Huffman code to assign binary codes to the directions that occur with the probabilities given below.

Up	Down	Left	Right
0.12	0.20	0.25	0.43

.32

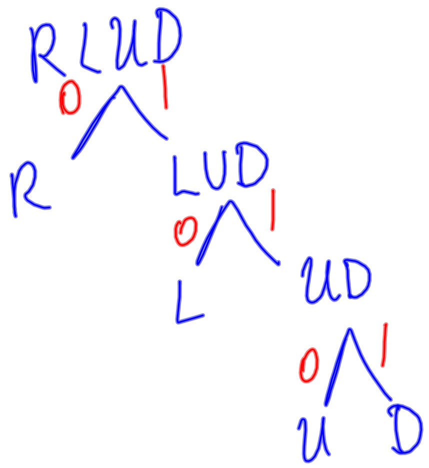
L	UD	R
.25	.32	.43

.57

R	LUD
.43	.57

RLUD

1



R → 0
L → 10
U → 110
D → 111

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Question 11 (5 points)

Use a decimation cipher with key 9 to encode the word CABLE.

	C	A	B	L	E
position	2	0	1	11	4
x 9	18	0	9	99	36
mod 26	18	0	9	21	10
Letter	S	A	J	V	K

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

Question 12 (5 points) Use a Caesar cipher with a shift of 3 to encode the word BINARY.

ELQDUB

0		2		4		6		8		10		12		14		16		18		20		22		24	
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Question 13 (6 points)
Use the Vigenere cipher with the key word PEN to encode BASEBALL.

	B	A	S	E	B	A	L	L
position	1	0	18	4	1	0	11	11
+	15	4	13	15	4	13	15	4
word	P	E	N	P	E	N	P	E
	16	4	31	19	5	13	26	15
mod 26	16	4	5	19	5	13	0	15
Letter	Q	E	F	T	F	N	A	P