

CHAPTER 7: PROBABILITY

7.1: Experiments, Sample Spaces and Events

An *experiment* is an activity with an observable result. Tossing coins, rolling dice and choosing cards are all probability experiments.

The result of the experiment is called the *outcome* or *sample point*. The set of all outcomes or sample points is called the *sample space* of the experiment.

Example

What is the sample space for flipping a fair coin? Rolling a 6-sided die?

An *event* is a subset of a sample space. That is, an event can contain one or more outcomes that are in the sample space.

Example

What are all possible events for the experiment of flipping a fair coin?

Example

How many events are possible when a six-sided die is rolled?

Example

What is the sample space for flipping a fair coin three times?

Find the event E where $E = \{x|x \text{ has exactly one head}\}$

Find the event E where $E = \{x|x \text{ has two or more heads}\}$

Find the event E where $E = \{x|x \text{ has more than 3 heads}\}$

A sample space in which each of the outcomes has the same chance of occurring is called a UNIFORM SAMPLE SPACE.

Example

A bowl contains the letters AGGIES. How many outcomes are in the uniform sample space?

What is the uniform sample space for rolling two fair six-sided dice?

1~1 2~1 3~1 4~1 5~1 6~1
 1~2 2~2 3~2 4~2 5~2 6~2
 1~3 2~3 3~3 4~3 5~3 6~3
 1~4 2~4 3~4 4~4 5~4 6~4
 1~5 2~5 3~5 4~5 5~5 6~5
 1~6 2~6 3~6 4~6 5~6 6~6

These sample spaces have all been finite. That is, we can list all the elements. An infinite sample space has to be described; you can't list all the elements:

Example

What is the sample space for the time spent working on a homework set?

Describe the event of spending between one and two hours on a homework set.

7.2 Definition of Probability

The probability of an event, $P(E)$ is a number between 0 and 1, inclusive. If $P(E) = 0$, then the event E is impossible. If $P(E) = 1$, then the event E is certain.

The *theoretical probability* of an event E occurring is based on the sample space S having equally likely outcomes. Then probability of the event E occurring is

$$P(E) = \frac{\text{number of outcomes in event } E}{\text{number of outcomes in the sample space}} = \frac{n(E)}{n(S)}$$

Example

Consider flipping a fair coin three times. Find the following probabilities:

- (a) Exactly one head is seen.
- (b) Two or more heads are seen.
- (c) More than 3 heads are seen.