LOGIC

A ***proposition*** or ***statement*** is a declarative sentence that can be classified as either true or false but not both.

Examples

- San Francisco is the capital of California. statement
- Be quiet! NO – a command
- Texas is east of California or it is west of California. statement
- When is the next exam? NO – question
- The Exam 1 results were good. A prop if "good" is defined
- $x + 1 = 4$ A prop if x is defined
- Miami Beach has 10^{10} grains of sand. A prop

A ***prime*** or ***simple*** proposition expresses one thought

p: Austin is the cap. of TX.

Join propositions with ***logical connectives*** to form compound propositions.

conjunction	AND	$\wedge$
negation	NOT	$\sim$
disjunction	OR	$\vee$

p: San Francisco is the capitol of California.
q: Austin is the capitol of Texas.

What is $p \wedge q$ in words? Is this proposition true or false?

SF is the cap of Calif and Austin is the cap of Texas. false

In general, The conjunction $p \wedge q$ is true only if both p and q are true

Show this in a ***truth table***,

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p: San Francisco is the capitol of California.
q: Austin is the capitol of Texas.

TRUE

What is $p \vee q$ in words? Is this proposition true or false?

SF is the cap of CA or Austin is the cap of TX

In general, The disjunction $p \vee q$ is false only if both p and q are false.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

This is called the ***inclusive disjunction***. This is also the mathematical ***or***.

Exclusive disjunction is $\veebar$. This is true only if exactly one of the two statements is true.

p: San Francisco is the capitol of California. F
q: Austin is the capitol of Texas. T

What is $p \veebar q$ in words? Is this proposition true or false?

Either SF is the cap of CA or Austin is the cap of TX (T)

r: Sacramento is the capitol of California. T

What is $r \veebar q$ in words? Is this proposition true or false?

Either Sac is the cap of CA or Austin is the cap of TX (F)

Show this in a ***truth table***,

p	q	$p \veebar q$
T	T	F
T	F	T
F	T	T
F	F	F

Negation: $\sim p$ means not p. Show this in a truth table:

p	$\sim p$
T	F
F	T

Write the following statements symbolically and find the truth table.

The car is blue or has a moon roof.

p: The car is blue

q: The car has a moon roof

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

The book is not red and the subject is history.

p: The book is red

q: The subject is history

p	q	∼p	∼p ∧ q
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

$\sim p \vee \sim q$

The sky is not blue or the grass is not purple.

p: The sky is blue

q: The grass is purple

p	q	∼p	∼q	∼p ∨ ∼q
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Define the following statements:

p: The student is a girl.

q: The student is a biology major.

r: The student is enrolled in a math class.

Write the following statements symbolically and find the truth table.

The student is a boy and is not a biology major or enrolled in a math class.

$\sim p \wedge (q \vee r)$

p	q	r	q∨r	~p	~p∧(q∨r)	
T	T	T	T	F	F	
T	T	F	T	F	F	
T	F	T	T	F	F	
T	F	F	F	F	F	
F	T	T	T	T	T	→ boy Bio major in math class
F	T	F	T	T	T	→ boy Bio may not in math class
F	F	T	T	T	T	Boy not Bio may in a math class
F	F	F	F	T	F	

A statement is a ***contradiction*** if the truth value of the statement is always false.

Example: Find the truth table for $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

A statement is a ***tautology*** if the truth value of the statement is always true.

Example: Find the truth table for $p \vee (\sim p \vee q)$

p	q	$\sim p$	$\sim p \vee q$	$p \vee (\sim p \vee q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T