Chapter1: Preview and Review

1.1 Preliminaries

NOTE this is not a complete list of material you are expected to know to be prepared for this course

Real number line:

Interval notation

(a,b)

and $[a,b]$

The **absolute value** of a real number a is $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

Example: Graph the function $y = |4x - 8|$

A vertical line has no slope (undefined). All other lines have a slope given by the equation

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

POINT-SLOPE form: $y - y_1 = m(x - x_1)$

SLOPE-INTERCEPT form: $y = mx + b$, b is the y-intercept

GENERAL form: $Ax + By + C = 0$

Example: A line has a slope of 2 and goes through the point (3, 4). What is the equation of the line? Graph the line and find the intercepts?

The quantities x and y are proportional ($y \propto x$) if they are linearly related ($y = mx$ or $y - b = mx$).

Radians and Degrees: $360^\circ = 2\pi$

A few common angles

$180^\circ = \pi$

$90^\circ = \frac{\pi}{2}$

$60^\circ = \frac{\pi}{3}$

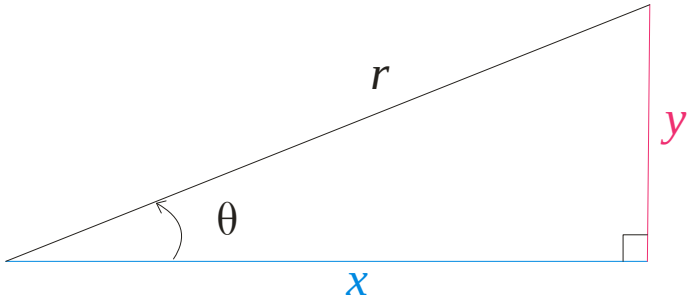
Example: Convert $\frac{17\pi}{12}$ to degrees and convert 50° to radians

Trigonometric Functions

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

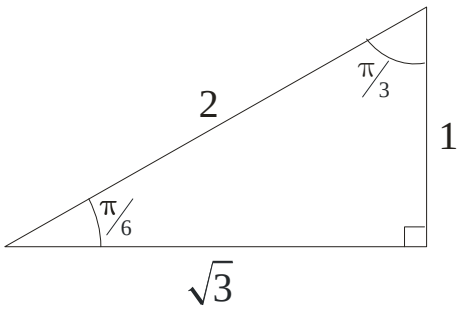
$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

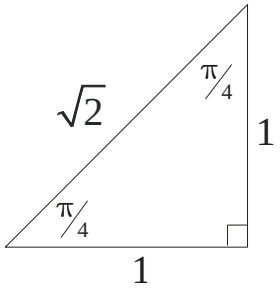
$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$



Pythagorean Theorem:

$x^2 + y^2 = r^2$





Example: (a) What is $\cos\frac{\pi}{3}$? (b) What is $\sin\frac{\pi}{4}$?

Some Formulas and Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

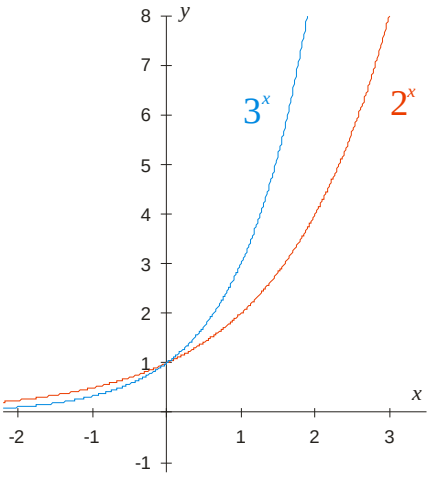
$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$\cos(x+y) = \cos x \cdot \cos y + \sin x \cdot \sin y$$

$f(x) = a^x, a > 0, a \neq 1$ is called an exponential function

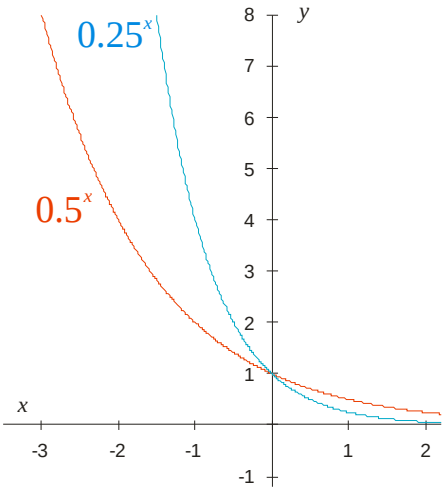
If $a > 1$, then this is exponential growth function where

- The domain is $(-\infty, \infty)$
- The range is $(0, \infty)$



If $0 < a < 1$, then this is exponential decay function where

- The domain is $(-\infty, \infty)$
- The range is $(0, \infty)$



Properties to remember: Given that a and b are positive,

$a^{x+y} = a^x a^y$ $\quad$ $\left(a^x\right)^y = a^{xy}$ $\quad$ $\left(ab\right)^x = a^x b^x$

$$\log_a x = y \quad \Leftrightarrow \quad a^y = x$$

Example: Evaluate the following

(a) $\log_2 64$

(b) $\log_6\left(\frac{1}{36}\right)$

(c) $\log_{16} 4$

If $x, y > 0$ then

$\log_a(xy) = \log_a x + \log_a y$ and $\log_a\left(x^y\right) = y\log_a x$

Two special logarithms: $\log_{10} x = \log x$ $\log_e x = \ln x = \ln x$

Two useful formulas:

$\log_a\left(a^x\right) = x$ $a^{\log_a x} = x$, for $x > 0$ $\log_a x = \frac{\ln x}{\ln a}$

Example: Express the given quantity as a single logarithm

(a) $\log_2 x + 5\log_2(x+1) + \frac{1}{2}\log_2(x-1)$

(b) $\ln x + a\ln y - b\ln z$

Example: Solve each equation for x

(a) $\log(x+1) = 3$

(b) $e^{3x-4} = 2$

(c) $\ln x + \ln(x-1) = 1$

(d) $\log x + \log(x+1) = \log 6$

A **complex number** is a number of the form

$$z = a + bi$$

where *a* and *b* are real numbers and $i^2 = -1$. The real part is *a* and the imaginary part is *b*.

Example: Find

(a) $(7-2i)-(9+6i)$

(b) $(7-2i)(9+6i)$

If $z = a + bi$ is a complex number, then its **conjugate** is $\bar{z} = a - bi$

Example: Let $z = 4-3i$. Compute

(a) $\bar{z}$

(b) $z\bar{z}$

A quadratic equation is an equation in the form $ax^2 + bx + c = 0$. Where *a*, *b*, and *c* are real numbers and $a \neq 0$. The solution to this equation is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve $3x^2 + 2x = -5$