3.3 Limits at Infinity

Let f be a function defined on some interval (a,∞). Then

$$\lim_{x\to\infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x to be sufficiently large.

Let f be a function defined on some interval $(-\infty,a)$. Then

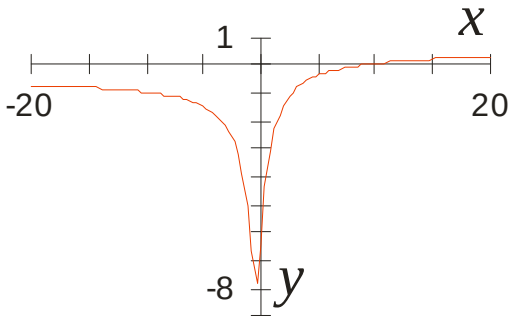
$$\lim_{x\to-\infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x to be sufficiently large negative.

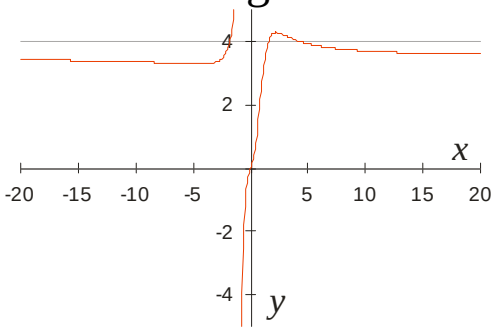
EXAMPLE 1

Determine the limits at infinity for the following functions

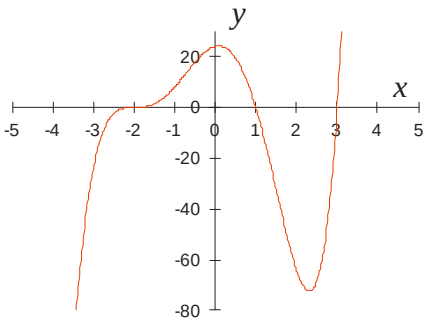
a)



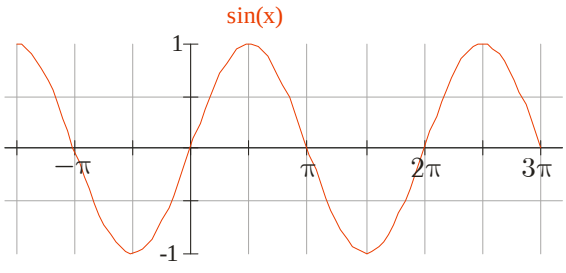
b)



c).



d)



If $r > 0$ is a rational number, then $\lim_{x\to\infty} \frac{1}{x^r} = 0$

If $r > 0$ is a rational number such that x^r is defined for all x, then $\lim_{x\to-\infty} \frac{1}{x^r} = 0$

Example: Evaluate the following limits and justify each step

a) $\lim_{x\to\infty} \dfrac{7x^3+4x}{2x^3-x^2+3}$

b) $\lim_{x\to\infty} \dfrac{x^3-1}{x^4+1}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{p(x)}{q(x)} = \begin{cases} 0 & \text{if } \deg(p) < \deg(q) \\ L \neq 0 & \text{if } \deg(p) = \deg(q) \\ DNE & \text{if } \deg(p) > \deg(q) \end{cases}$$

Example: Find the following limits

a) $\lim_{x\to\infty}\sqrt{\dfrac{2x^2-1}{x+8x^2}}$

b) $\lim_{x\to-\infty}\dfrac{6t^2+5t}{(1-t)(2t-3)}$

c) $\lim_{x\to-\infty}\dfrac{x^4+2x+3}{x\left(x^2-1\right)}$

d) $\lim_{x\to-\infty}\dfrac{\sqrt{x^2+4x}}{4x+1}$

e) $\lim_{x\to\infty}\left(\sqrt{x^2+3x+1}-x\right)$

The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = L \quad \text{or} \quad \lim_{x\to-\infty} f(x) = L$$

Example: Find the horizontal and vertical asymptotes of each curve.

a) $y = \dfrac{x^2+4}{x^2-1}$

b) $y = \dfrac{x-9}{\sqrt{4x^2+3x+2}}$

$$\lim_{x\to\infty} e^{-x} = 0$$

Example: Given $N(t) = \dfrac{50}{1+3e^{-2t}}$, find $\lim\limits_{t\to\infty} N(t)$ and graph $N(t)$