

3.4 The Sandwich Theorem and Some Trigonometric Limits

Sandwich Theorem: If $f(x) \leq g(x) \leq h(x)$ for all x in an open interval that contains a (except possibly at a) and

$$\lim_{x \rightarrow a} f(x) = L = \lim_{x \rightarrow a} h(x)$$

then

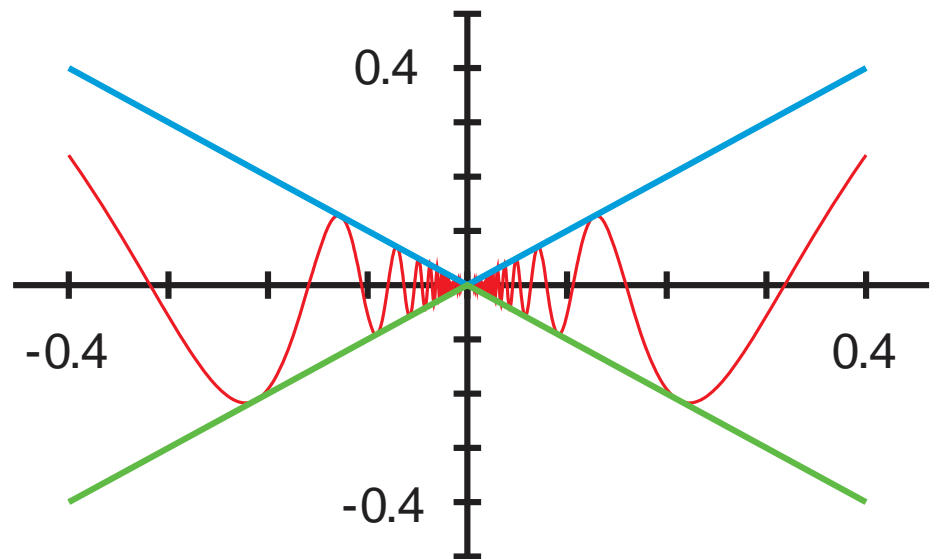
$$\lim_{x \rightarrow a} g(x) = L$$

Example: What is $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$?

$$f(x) = -|x|$$

$$g(x) = x \sin\left(\frac{1}{x}\right)$$

$$h(x) = |x|$$



Theorem: $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

Corollary: $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} = 0$

Example: Find the following limits

$$\lim_{x \rightarrow 0} \cos(\sin x)$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{3x}$$

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{2x}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 8x}$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x}$$

$$\lim_{x \rightarrow 0} \frac{\tan^2 4x}{x^2}$$