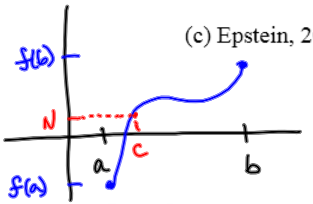

3.5 Properties of Continuous Functions

Intermediate Value Theorem

Suppose f is continuous on the closed interval $[a,b]$ and let N be any number strictly between $f(a)$ and $f(b)$. Then there exists a number c in (a,b) such that $f(c)=N$.

Example: Use the Intermediate Value Theorem to show there is a root of the given equation in the given interval.

a) $x^5-2x^4-x-3=0,\quad (2,3)$

$f(x)=x^5-2x^4-x-3$ is a polynomial and therefore continuos on $\mathbb{R}$

$f(2)=-5$ and $f(3)=75$

Since there is a sign change on $[2,3]$, there is a point in $(2,3)$ where $f(x)=0$

$-\sqrt{x+1}$ $-\sqrt{x+1}$

b) $x^2=\sqrt{x+1},\quad (1,2)$

$f(x)=x^2-\sqrt{x+1}$. $f(x)$ is cont on $x\geq -1$ so IVT says there is a soln to $x^2-\sqrt{x+1}=0$ if there is a sign change

$f(1)=1^2-\sqrt{1+1}=1-\sqrt{2}<0$

$f(2)=2^2-\sqrt{2+1}=4-\sqrt{3}>0$

sign change found ⇒ $f(x)=x^2-\sqrt{x+1}=0$ has a soln on $(1,2)$

~~*Example:* Use the Intermediate Value Theorem to show that there is a positive number c such that $c^2=2$.~~

In which of the intervals below does $y = -x^3 + 4x^2 - 5x + 3$ have a root?

(A) [-1, 0] (B) [0, 1] (C) [1, 2] [✓ (D) [2, 3]] (E) None of these contain a root.

$y(-1)=-(-1)^3+4(-1)^2-5(-1)+3=13$

$y(0)=3$

$y(1)=1$

$y(2)=1$

$y(3)=-3$

$\left.\begin{matrix} y(2)=1 \\ y(3)=-3 \end{matrix}\right\}$ Sign change