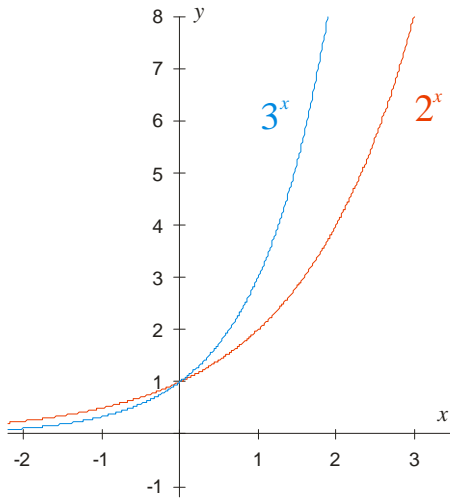


## 4.6 Derivatives of Exponential Functions

*Example:* Given  $f(x) = 2^x$  and  $g(x) = 3^x$ , estimate  $f'(0)$  and  $g'(0)$  using a table of values.



Definition:  $e$  is the number such that  $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$

Therefore:  $\frac{d}{dx} e^x = e^x$  and so  $\frac{d}{dx} e^{g(x)} = g'(x)e^{g(x)}$

*Example:* Find the derivatives of the following functions

(a)  $y = \sqrt{e^x + x}$

(b)  $f(x) = e^{-5x} \cos(3x)$

(c)  $g(x) = e^{x \sin x}$

(d)  $f(x) = a^x$

(e)  $f(x) = 3^{x^3-1}$

*Example:* Find the equation of the tangent line to the graph of  $2e^{xy} = x + y$  at the point  $(0, 2)$

*Example:* For what value(s) of  $r$  does  $y = e^{rx}$  satisfy the differential equation  $y + y' = y''$ ?

*Example:* Suppose a bacterial colony grows in such a way that at time  $t$  the population size is  $N(t) = N(0)2^t$  where  $N(0)$  is the population size at time  $t = 0$ . Find the per capita growth rate.

*Example:* Suppose  $W(t)$  denotes the amount of a radioactive material left after time  $t$  days. Assume that the radioactive decay rate of the material is 0.2/day. Find the differential equation for  $W(t)$ .