4.7 Derivatives of Inverse Functions

Theorem: If $f(x)$ is a one-to-one differentiable function with inverse $g(x) = f^{-1}(x)$ and $f'\left[f^{-1}(x)\right] \neq 0$, then $g(x)$ is differentiable and

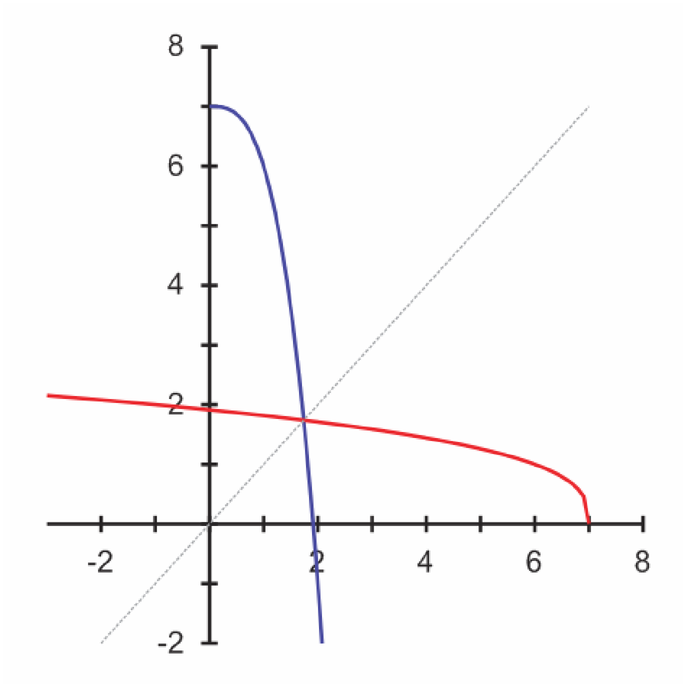
$$\frac{d}{dx} f^{-1}(x) = g'(x) = \frac{1}{f'\left(g(x)\right)}$$

Example: Suppose g is the inverse of f and $f(2) = 3$, $f'(2) = 7$, $f(3) = 4$, and $f'(3) = \frac{1}{2}$, find $g'(3)$.

Example: Suppose g is the inverse of f. Find $g'(4)$ if $f(x) = 3 + x + e^x$.

Example: Suppose g is the inverse of f. Find $g'(2)$ if $f(x) = \sqrt{x^3 + x^2 + x + 1}$.

Example: Let $f(x) = -x^3 + 7, x \geq 0$. Find the derivative of the inverse at $x = -1$. Note that $f(2) = -1$.



Example: $y = \ln x$, find dy/dx

In general, $\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}$. Other useful formulas are

$$\frac{d}{dx}\ln|x| = \frac{1}{x} \qquad \frac{d}{dx}\log_a g(x) = \frac{g'(x)}{g(x)\ln a} \qquad \frac{d}{dx}a^{g(x)} = a^{g(x)}\ln a \, g'(x)$$

Example: Find the following derivatives

(a) $y = \cos\left(\ln x + \pi\right)$

(b) $f(x) = \ln\left(x + \ln x\right)$

(c) $g(t) = \ln\sqrt{\dfrac{3t+2}{3t-2}}$

(d) $h(y) = \ln\left(y^3 \sin y\right)$

(e) $y = \ln\left|\tan 2x\right|$

(f) $y = \tan^5 x + 5^{\tan x}$

Logarithmic Differentiation is useful in for the following cases

- Functions of the form $y = f(x)^{g(x)}$
- Functions with products or quotients you wish to split

EXAMPLE 4

Find the following derivatives

(a) $f(x) = (\sin x)^{\cos x}$ (b) $y = x^x$ (c) $g(x) = x^{2/5}(x^2+8)^4 e^{x^2+x}$