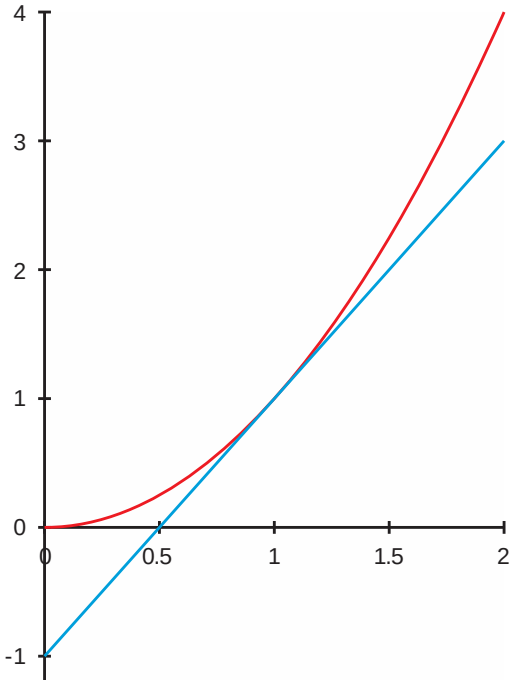
4.8 Linear Approximation and Error Propagation

The graph of $y = x^2$ is shown along with the tangent line at $x = 1$.



What is the equation of the tangent line at $x = 1$ for $y = x^2$?

Assume that $y = f(x)$ is differentiable at $x = a$, then

$$L(x) = f(a) + f'(a)(x - a)$$

is the **tangent line approximation** or the **linearization** of f at $x = a$.

Example:

(a) Find the linear approximation of $f(x) = \frac{1}{x}$ at $x = 4$

(b) Use the linearization to find the approximate value of $\frac{1}{3.9}$.

Example: Find the linear approximation of $f(x)=\dfrac{1}{3-2x}$ at $a=2$.

Example: Find the linear approximation of $f(x)=e^{2x}$ at $a=0$ and use it to approximate $e^{-0.4}$.

Example: Find the linear approximation of $f(x)=\sin x$ at $x=0$ and use it to approximate sin(0.1).

We can use our linear approximation to see how measurement errors are propagated when used in formulas. Suppose that x_0 is the true value of an observation and x is the measured value. The **absolute error** (or tolerance) in the measurement is $|\Delta x| = |x - x_0|$.

The **relative error** is $|\Delta x / x_0|$ and the **percentage error** is $100|\Delta x / x_0|$

Example: The radius of a circular disk is given to be 24 cm with a maximum error in measurement of 0.2 cm. Find the absolute and relative error in the area of the disc.

Example: The circumference around the middle of a sphere is measured to be 40 cm, with a possible error of ±1 cm. Estimate the possible error in the volume of the sphere.