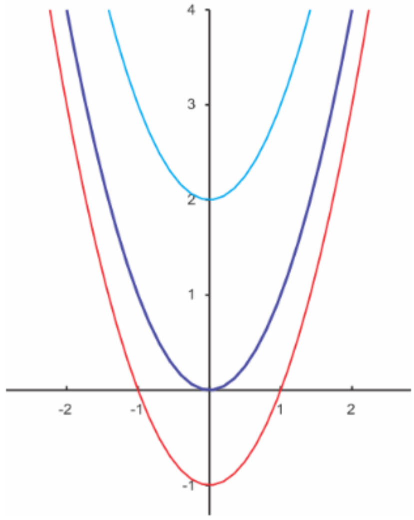
5.8: Antiderivatives

The derivative of what function $F(x)$ will give the function $f(x) = 2x$?

Since the derivative of x^2 is $2x$, we can say the function $F(x) = x^2$ is the antiderivative of $2x$.

Note however, that $\frac{d}{dx}\left(x^2 + c\right) = 2x$



The function $F(x)$ an **antiderivative** of $f(x)$ if $F'(x) = f(x)$

Function	Antiderivative
k	$kx + c$
$x^n,\ n \neq -1$	$\frac{x^{n+1}}{n+1} + c$
x^{-1}	$\ln\lvert x\rvert + c$
e^x	$e^x + c$
$f(x) + g(x)$	$F(x) + G(x)$

Function	Antiderivative
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
$\sec^2 x$	$\tan x + c$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x + c$
$\frac{1}{x^2+1}$	$\arctan x + c$

Example: Find the most general antiderivative for the following functions

(i) $f(x) = x^3 - 4x^2 + 17$

$F(x) = \frac{x^{3+1}}{3+1} - 4\frac{x^{2+1}}{2+1} + 17x + C = \frac{x^4}{4} - \frac{4}{3}x^3 + 17x + C$

(ii) $f(x) = \sqrt[3]{x^2} - \sqrt{x^3} = x^{2/3} - x^{3/2}$

$F(x) = \frac{x^{2/3+1}}{2/3+1} - \frac{x^{3/2+1}}{3/2+1} + C = \frac{3}{5}x^{5/3} - \frac{2}{5}x^{5/2} + C$

(iii) $f(x) = \frac{x + x^2 - 1}{x^3} = \frac{x}{x^3} + \frac{x^2}{x^3} - \frac{1}{x^3} = x^{-2} + x^{-1} - x^{-3}$

$F(x) = \frac{x^{-2+1}}{-2+1} + \ln|x| - \frac{x^{-3+1}}{-3+1} = \frac{-1}{x} + \ln|x| + \frac{1}{2x^2} + C$

(iv) $f(x) = e^x + \frac{4}{\sqrt{1-x^2}}$ $F(x) = e^x + 4\arcsin(x) + C$

Example: Find $f(x)$ given that

(i) $f'(x)=12x^2-24x+1$ and $f(1)=-2$ what is c? c = 5

$f(x)= 4x^3-12x^2+x+c$

$f(1) = -2 = 4(1)^3-12(1)^2+(1)+c$

$f(x)= 4x^3-12x^2+x+5$

(ii) $f''(x)=3e^x+4\sin x$, $f(0)=1$, and $f'(0)=2$

$f'(x)= 3e^x-4\cos x+c$

$f'(0)=2 = 3e^0-4\cos 0+c \Rightarrow c=3$

$f'(x)= 3e^x-4\cos x+3$

$f(x)= 3e^x-4\sin x+3x+d$

$f(0)=1 = 3e^0-4\sin 0+3(0)+d \Rightarrow d=-2$

$f(x)= 3e^x-4\sin x+3x-2$

Example: A particle is moving with acceleration $a(t)=3t+8$ m/s^2. Find the position $s(t)$ of the object at time t if we know $s(0)=1$ and $v(0)=-2$

$S(10) = \underline{881}$ m $\quad a(t)=3t+8 = v'(t) = s''(t)$

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$v(t)= 3\frac{t^2}{2}+8t+c$, $v(0)=-2=c \Rightarrow v(t)=\frac{3}{2}t^2+8t-2$

$s(t)= \left(\frac{3}{2}\right)\frac{t^3}{3}+8\frac{t^2}{2}-2t+d = \frac{t^3}{2}+4t^2-2t+d$, $s(0)=1$

$s(t)=\frac{t^3}{2}+4t^2-2t+1$

Example: A stone is thrown downward from a 450m tall building at a speed of 5 m/s. Find a formula for the distance of the stone above ground.

v_0

on Earth $a(t) = -9.8\ m/s^2 = v'(t) \Rightarrow v(t) = -9.8t + v_0 = -9.8t + 5 = s'(t)$

$s(t) = -9.8\frac{t^2}{2} - 5t + s_0 = -4.9t^2 - 5t + 450$ pos in m, time in sec

Example: A car braked with constant deceleration of 40 ft/s². The skid marks produced were 160 ft before the car came to a stop. How fast was the car traveling when the brakes were first applied?

$a(t) = -40 = v'(t)$

$v(t) = -40t + v_0 = s'(t)$

$s(t) = -\frac{40t^2}{2} + v_0 t + s_0$

$v(t_F) = 0 = -40t_F + v_0$

$v_0 = 40t_F$

$\Rightarrow s(t) = -20t^2 + v_0 t$

$s(t_F) = 160 = -20t_F^2 + v_0 t_F$

$160 = -20t_F^2 + 40t_F \cdot t_F = 20t_F^2$

$t_F^2 = \frac{160}{20} = 8 \Rightarrow t_F = \sqrt{8}$ sec

$v_0 = 40\sqrt{8}$ ft/sec (77 mph)