7.1: The Substitution Rule

Let $f(x)=e^{x^2}$, then $f'(x)=2xe^{x^2}$. What does that mean for $\int xe^{x^2}\,dx$?

If you have an integrand of the form $f'[g(x)]\cdot g'(x)$, then

$$\int f'[g(x)]g'(x)\,dx = f[g(x)]+C$$

General Indefinite Integral Formulas

$$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1}+C, \quad n\neq -1$$

$$\int e^{f(x)} f'(x)\,dx = e^{f(x)}+C$$

$$\int \frac{1}{f(x)} f'(x)\,dx = \ln|f(x)|+C$$

$$\int f'(x)\sin[f(x)]\,dx = -\cos[f(x)]+C$$

$$\int f'(x)\cos[f(x)]\,dx = \sin[f(x)]+C$$

$$\int f'(x)\sec^2[f(x)]\,dx = \tan[f(x)]+C$$

Example: Evaluate $\int (x^2+10x+7)^4(2x+10)\,dx$

Integration by Substitution Rule for Indefinite Integrals:

$$\int f\left[g(x)\right]g'(x)\,dx = \int f(u)\,du \text{ where } u = g(x)$$

Example: Evaluate the following integrals

(i) $\int 3x^9\sqrt{4x^{10}-7}\,dx$ (ii) $\int \frac{\ln t}{t}\,dt$ (iii) $\int \tan x\,dx$ (iv) $\int \frac{x}{\sqrt{x-9}}\,dx$

Integration by Substitution Rule for Definite Integrals:

$$\int_a^b f\left[g(x)\right]g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du \text{ where } u = g(x)$$

Example: Evaluate the following integrals

(i) $\int_{-1}^{4} x\sqrt{x+5}\,dx$ (ii) $\int_{0}^{\pi/3} \frac{\sin x}{\cos^2 x}\,dx$ (iii) $\int_{\ln 4}^{\ln 7} \frac{e^x}{\left(e^x-3\right)^2}\,dx$ (iv) $\int_{1}^{2} x^5\sqrt{x^3+2}\,dx$