Calculating Limits Using the Limit Laws (2.3)

Suppose that *c* is a constant, *n* is a positive integer, and the limits

$\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$

exist. Then the following laws can be used to calculate limits.

Note that *n* is a positive integer

1. $\lim_{x\to a}\left[f(x)+g(x)\right]=\lim_{x\to a} f(x)+\lim_{x\to a} g(x)$
2. $\lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a} f(x)-\lim_{x\to a} g(x)$
3. $\lim_{x\to a}\left[cf(x)\right]=c\lim_{x\to a} f(x)$
4. $\lim_{x\to a}\left[f(x)\cdot g(x)\right]=\lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$
5. $\lim_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x)\neq 0$
6. $\lim_{x\to a}\left[f(x)\right]^n=\left[\lim_{x\to a} f(x)\right]^n$
7. $\lim_{x\to a} c=c$
8. $\lim_{x\to a} x=x$
9. $\lim_{x\to a} x^n=a^n$
10. $\lim_{x\to a}\sqrt[n]{x}=\sqrt[n]{a}$ (if *n* is even, assume $a>0$)
11. $\lim_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to a} f(x)}$ (if *n* is even, assume $\lim_{x\to a} f(x)>0$)

EXAMPLE 1

Given that $\lim_{x\to 2} f(x)=3$, find the following using the limit laws.

a) $\lim_{x\to 2}\left[\left(f(x)\right)^2-2x+4\right]$

b) $\lim_{x\to 2}\dfrac{\sqrt{f(x)+6}}{2f(x)+x+11}$

EXAMPLE 2

Find the following limits, if they exist

a) $\lim_{x\to -2}\left(x^2+x+1\right)^5$

b) $\lim_{x\to -3}\dfrac{x^2-x-12}{x+3}$

c) $\lim_{x\to 64}\left(\sqrt[3]{x}+3\sqrt{x}\right)$

d) $\lim_{x\to -1}\dfrac{x^2-x-3}{x+1}$

e) $\lim_{h\to 0}\dfrac{(2+h)^3-8}{h}$

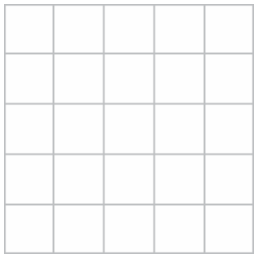
f) $\lim_{x\to 1.5^+}\left(\sqrt{3+2x}+x\right)$

g) $\lim_{x\to 4}\dfrac{x+1}{(x-4)^2}$

h) $\lim_{t\to -2}\dfrac{t^3-t^2-t+10}{t^2+3t+2}$

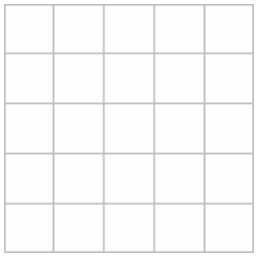
Some special functions are

Absolute value $= |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



Greatest Integer $= [\![x]\!]$

= the largest integer that $\leq$ to x



EXAMPLE 3

Find the following limits, if they exist.

a) $\lim\limits_{x \to -4} \dfrac{|x+4|}{x+4}$

b) $\lim\limits_{x \to -2} [\![x]\!]$

c) $\lim\limits_{x \to 0} f(x)$ and $\lim\limits_{x \to 2} f(x)$ given that $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ 4 - x & \text{if } x > 2 \end{cases}$

EXAMPLE 4

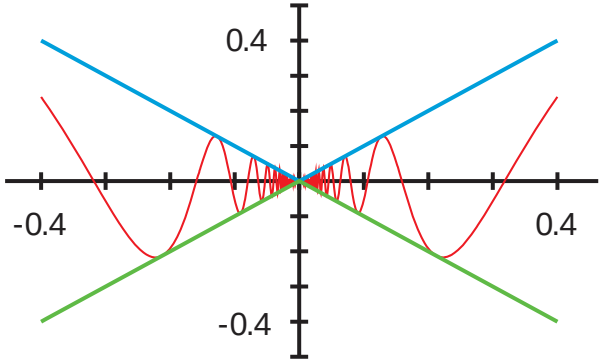
The position of a moving particle at time t, $0 \leq t < 5$ is given by $\boldsymbol{r}(t) = \left\langle \dfrac{2t-10}{t-5}, \dfrac{t-5}{t^2-4t-5} \right\rangle$. What is the anticipated position of the particle at $t = 5$?

Theorem:
If $f(x) \le g(x)$ for all x in an open interval that contains a (except possibly at a) and the limits of f and g both exist as x approaches a, then $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$

$g(x) = x\sin\frac{1}{x}$

$f(x) = |x|$

$h(x) = -|x|$



To find $\lim_{x\to 0} x\sin\frac{1}{x}$, we can use the Squeeze Theorem.

If $f(x) \le g(x) \le h(x)$ for all x in an open interval that contains a (except possibly at a) and $\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x)$ then $\lim_{x\to a} g(x) = L$

$\lim_{x\to 0} x\sin\frac{1}{x} =$

EXAMPLE 5
Given that $3x \le f(x) \le x^3 + 2$ for all the x in the interval (0, 3), find $\lim_{x\to 1} f(x)$

EXAMPLE 6

Is there a value or values of a, such that $\lim_{x\to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exists? If so, what is the value of the limit?