Maximum and Minimum Values (Section 5.2)

Absolute Extreme values of f

- A function *f* has an absolute (or global) maximum at *c* if $f(c) \geq f(x)$ for all *x* in the domain of *D*. The number $f(c)$ is called the maximum value of *f* on *D*.
- A function *f* has an absolute (or global) minimum at *d* if $f(d) \leq f(x)$ for all *x* in the domain of *D*. The number $f(d)$ is called the minimum value of *f* on *D*.

Local Extreme values of f

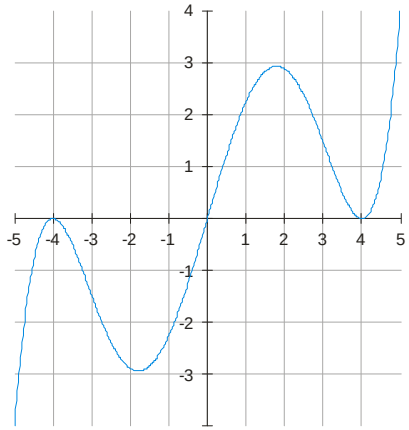
- A function *f* has a local (or relative) maximum at *c* if $f(c) \geq f(x)$ when *x* is near *c*.
- A function *f* has a local (or relative) minimum at *d* if $f(d) \leq f(x)$ when *x* is near *c*.

The Extreme Value Theorem

If *f* is continuous on a closed interval $[a,b]$, the f attains an absolute maximum value of $f(c)$ and an absolute minimum value $f(d)$ at some numbers *c* and *d* in $[a,b]$

EXAMPLE 1

The graph of the function *f* with domain $[-5,5]$ is shown.



(a) What are the absolute extrema?

(b) What are the local extrema?

EXAMPLE 2

Find all absolute and local extrema for the following functions

$f(x) = 1 - x^2,\ 0 \leq x \leq 1$

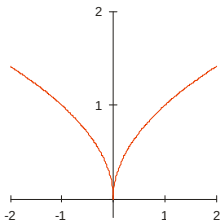
$f(x) = 1 - x^2,\ -1 \leq x \leq \frac{1}{2}$

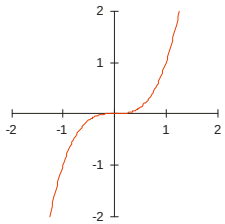
$$f(x)=\frac{1}{x},\ 0<x<1$$

$$f(x)=\begin{cases} x^2 & \text{for } -1\le x<0 \\ 2-x^2 & \text{for } 0\le x\le 1 \end{cases}$$

Fermat's Theorem

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c)=0$.





Critical Numbers

A critical number (or value) of a function f is a number c in the domain of f such that either $f'(c)=0$ or $f'(c)$ does not exist.

EXAMPLE 3

Find all critical numbers for the following functions

$$f(x)=x^3+6x^2+3x-1$$

$$f(x)=\left|x^2-1\right|$$

$$f(x)=\frac{x+1}{x^2+x+1}$$

$$f(x)=x+\sin x$$

$$f(x)=xe^{2x}$$

$$f(x)=\sqrt[3]{x^2-x}$$

Finding Absolute Extrema

To find the absolute extrema of a continuous function f on a closed interval $[a,b]$,

1. Find all critical values c_1, c_2, etc of f in the interval (a,b)
2. Find the values of $f(c_1)$, $f(c_2)$ etc along with $f(a)$ and $f(b)$
3. The absolute maxima is the largest of the values found in steps 1 and 2.
4. The absolute minima is the smallest of the values found in steps 1 and 2.

EXAMPLE 4

Find the absolute extrema for the following functions

i. $f(x) = x^3 - 12x + 1, \quad [-3,5]$

ii. $f(x) = -x^3 + 27x + 1, \quad [0,4]$

iii. $f(x) = x - 2\cos x, \quad [0,\pi]$

iv. $f(x) = x - 2\cos x, \quad [-\pi,\pi]$

v. $f(x) = \dfrac{x}{x+1}, \quad [1,2]$

vi. $f(x) = \dfrac{x}{x+1}, \quad [-2,2]$

vii. $f(x) = \dfrac{\ln x}{x}, \quad [1,3]$