Antiderivatives (Section 5.7)

The derivative of what function $F(x)$ will give the function $f(x)=2x$? Since the derivative of x^2 is $2x$, we can say the function $F(x)=x^2$ is the antiderivative of $2x$.

Antiderivatives
The function $F(x)$ an antiderivative of $f(x)$ if $F'(x)=f(x)$

TABLE OF ANTIDERIVATIVES

Function	Antiderivative		Function	Antiderivative
k	$kx+c$		$\sec x \tan x$	$\sec x + c$
$x^n,\ n \neq -1$	$\dfrac{x^{n+1}}{n+1}+c$		$\sec^2 x$	$\tan x + c$
x^{-1}	$\ln\lvert x\rvert + c$		$-\csc^2 x$	$-\cot x + c$
e^x	$e^x + c$		$-\csc x \cot x$	$-\csc x + c$
$\cos x$	$\sin x + c$		$\dfrac{1}{x^2+1}$	$\arctan x + c$
$\sin x$	$-\cos x + c$		$\dfrac{1}{\sqrt{1-x^2}}$	$\arcsin x + c$

EXAMPLE 1
Find the most general antiderivative for the following functions

(i) $f(x)=x^3-4x^2+17$

(ii) $f(x)=\sqrt[3]{x^2}-\sqrt{x^3}$

(iii) $f(x)=\dfrac{x+x^2-1}{x^3}$

(iv) $f(x)=e^x+\dfrac{4}{\sqrt{1-x^2}}$

EXAMPLE 2

Find $f(x)$ given that

(i) $f'(x) = 12x^2 - 24x + 1$ and $f(1) = -2$

(ii) $f''(x) = 3e^x + 4\sin x$, $f(0) = 1$, and $f'(0) = 2$

EXAMPLE 3

A particle is moving with acceleration $a(t) = 3t + 8$ m/s^2. Find the position $s(t)$ of the object at time t if we know $s(0) = 1$ and $v(0) = -2$

EXAMPLE 4

A stone is thrown downward from a 450m tall building at a speed of 5 m/s. Find a formula for the distance of the stone above ground.

EXAMPLE 5

A car braked with constant deceleration of 40 ft/s^2. The skid marks produced were 160 ft before the car came to a stop. How fast was the car traveling when the brakes were first applied?

A vector function $\boldsymbol{R}(t) = X(t)\boldsymbol{i} + Y(t)\boldsymbol{j} = \langle X(t), Y(t) \rangle$ is called the antiderivative of $\boldsymbol{r}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j} = \langle x(t), y(t) \rangle$ on an interval I if $\boldsymbol{R}'(t) = \boldsymbol{r}(t)$ for all t in I.

That is, $X'(t) = x(t)$ and $Y'(t) = y(t)$.

The most general antiderivative of $\boldsymbol{r}$ on I is $\boldsymbol{R}(t) + \boldsymbol{C}$ where $\boldsymbol{C}$ is an arbitrary constant vector.

EXAMPLE 6

A projectile is fired from a position 200m above the ground with an initial speed of 500m/s and an angle of elevation of 30° above the horizontal. Find a vector equation for the position of the projectile at time t in seconds.