EXAM 3 MATERIAL

1. Definitions to absolutely know

(1) equipotence
(2) finite, countably infinite, countable set
(3) partial order
(4) lattice
(5) order complete lattice
(6) isotone self-map
(7) limit superior
(8) limit inferior
(9) Cauchy sequences
(10) metric, metric spaces
(11) norm, normed linear spaces
(12) open and closed sets
(13) convergence of real sequences
(14) Cauchy sequences
(15) complete metric spaces
(16) Banach spaces
(17) total boundedness
(18) compactness
(19) open covers
(20) connected/disconnected sets
(21) continuity at a point, global continuity
(22) uniformly continuous functions
(23) Lipschitz functions
(24) pointwise convergence of sequences of functions
(25) uniform convergence of sequences of functions

2. Theorems to absolutely know

(1) Cantor's theorem.
(2) Tarski's fixed-point theorem.
(3) Cantor-Schröder-Berstein theorem.
(4) Bolzano-Weierstraß theorem.
(5) Hölder's inequality.
(6) Minkowski's inequality.
(7) Sequential characterization of closed sets.
(8) Banach's contraction principle.
(9) Heine-Borel theorem (multidimensional version).
(10) Generalized Intermediate Value theorem.
(11) Dini's theorem.
(12) Pólya's theorem.
(13) Weierstraß approximation theorem.
(14) Generalized Mean Value theorem.
(15) Darboux theorem.
(16) Weierstraß M-test.
(17) Helly's first theorem.
(18) Riemann criterion.
(19) Helly's second theorem.
(20) Dirichlet's theorem.

3. Proofs to absolutely know

(1) Tarski's fixed-point theorem.
(2) Characterizations of limit superior and limit inferior (Thm 2.2.7).
(3) Subsequences converging to limit superior and limit inferior (Thm 2.2.10).
(4) Hölder's inequality.
(5) Minkowski's inequality.
(6) Completeness of ℓ^p spaces (Example 3.3.9).
(7) Completeness and closed subsets (Prop 3.3.10).
(8) Banach's contraction principle.
(9) Total boundedness and Cauchy subsequences (Thm 3.4.6).
(10) Characterization of compactness in terms of convergent subsequences (Thm 3.5.4).
(11) Characterization of compactness in terms of open covers (Thm 3.5.6).
(12) Total boundedness of closed and bounded subsets of ℓ_n^∞ (case I in Thm 3.5.9).
(13) A continuous image of a connected set is connected (Thm 3.7.6).
(14) A continuous image of a compact set is compact (Thm 3.7.8).
(15) Sequential characterization of uniform continuity (Thm 3.8.6).
(16) Automatic uniform continuity of continuous functions on compact sets (Thm 3.8.9).
(17) A uniform limit of continuous functions is continuous (Thm 3.9.5).
(18) Weierstraß M-test (Thm 4.2.1).
(19) Cantor's diagonal extraction principle (Thm 4.4.10).
(20) Riemann criterion (Thm 4.5.10).