REAL ANALYSIS MATH 608
HOMEWORK #1

Problem 1.

(1) *Show that a topological space is* T_1 *if and only if every singleton is closed.*
(2) *Show that the cofinite topology on an infinite set is* T_1 *but not* T_2.

Problem 2.

(1) *Show that A is nowhere dense in* $(X,\mathcal{T})$ *if and only if for every nonempty open set U,* $\overline{A} \cap U \neq U$.
(2) *Interpret the statement in* (1) *in the relative topology.*

Problem 3. *Show that a separable metric space is second countable.*

Problem 4 (The Sorgenfrey line or upper limit topology)**.** *Let* $\mathscr{C} \overset{\text{def}}{=} \{(a,b] : -\infty < a < b < \infty\}$.

(1) *Show that* $\mathscr{C}$ *is a base for a topology, denoted* $\mathcal{T}_u$*, on* $\mathbb{R}$ *such that the sets in* $\mathscr{C}$ *are clopen (i.e. open and closed).*
(2) *Show that* $(\mathbb{R},\mathcal{T}_u)$ *is first countable, separable, but not second countable.*

Problem 5. *Let* (X,τ) *be a topological space and let* $acc(A)$ *denote the set of accumulation points of A. Show that*

(1) $\overline{A} = A \cup acc(A)$
(2) $x \in acc(A)$ *if and only if there exists a net in* $A \setminus \{x\}$ *that converges to x.*