MATH 300
Practice EXAM II
Duration: 75 minutes

LAST NAME: ____________________________FIRST NAME: ____________________________

SECTION NUMBER: ______________

DIRECTIONS:

1. The use of a calculator, laptop, computer, book, or lecture notes is prohibited.
2. *Show all your work neatly and concisely.*

THE AGGIE CODE OF HONOR
"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: ____

Exercise	Points Awarded	Points
1-10		40
11		12
12		12
13		8
14		16
15		12
TOTAL		100

1. Exercise 1. (4 points) Which of the following statements is a proposition.
   (a) $x + 1 = 0$.
   (b) $(\exists x)[x + 1 = 0]$.
   (c) none of the above.
2. Exercise 2. (4 points) Which of the following statements is a proposition.
   (a) $(\forall x)[x^2 + y^2 \neq 0]$.
   (b) $(\exists y)(\forall x)[x^2 + y^2 \neq 0]$.
   (c) none of the above.
3. Exercise 3. (4 points) Which of the following statements is a predicate.
   (a) $(\forall x)[x^2 + y^2 \neq 0]$.
   (b) $(\forall x)[(x > 1) \implies (x^2 > 4)]$.
   (c) none of the above.
4. Exercise 4. (4 points) Which of the following statements is a predicate.
   (a) $(\forall x)[x^2 + 1 \neq 0]$.
   (b) $(x > 2) \implies (y^4 + x > 0)$.
   (c) none of the above.
5. Exercise 5. (4 points) Let X and Y be sets. By definition $X \cup Y$ is the set
   (a) $\{z \mid (z \notin X) \wedge (z \in Y)\}$,
   (b) $\{z \mid (z \in X) \vee (z \in Y)\}$,
   (c) $\{z \mid (z \notin X) \vee (z \notin Y)\}$,
   (d) $\{z \mid (z \in X) \wedge (z \in Y)\}$,
   (e) $\{z \mid (z \in X) \wedge (z \notin Y)\}$,
   (f) none of the above.
6. Exercise 6. (4 points) Let X and Y be sets. By definition $X \cap Y$ is the set
   (a) $\{z \mid (z \notin X) \wedge (z \in Y)\}$,
   (b) $\{z \mid (z \in X) \vee (z \in Y)\}$,
   (c) $\{z \mid (z \notin X) \vee (z \notin Y)\}$,
   (d) $\{z \mid (z \in X) \wedge (z \in Y)\}$,
   (e) $\{z \mid (z \in X) \wedge (z \notin Y)\}$,
   (f) none of the above.

7. Exercise 7. (4 points) Let X and Y be sets. By definition $X - Y$ is the set
   (a) $\{z \mid (z \notin X) \wedge (z \in Y)\}$,
   (b) $\{z \mid (z \in X) \vee (z \in Y)\}$,
   (c) $\{z \mid (z \notin X) \vee (z \notin Y)\}$,
   (d) $\{z \mid (z \in X) \wedge (z \in Y)\}$,
   (e) $\{z \mid (z \in X) \wedge (z \notin Y)\}$,
   (f) none of the above.

8. Exercise 8. (4 points) Let X and Y be sets. By definition $Y \times X$ is the set
   (a) $\{(y, x) \mid (x \in X) \vee (y \in Y)\}$,
   (b) $\{(x, y) \mid (x \in X) \vee (y \in Y)\}$,
   (c) $\{(x, y) \mid (x \in X) \wedge (y \in Y)\}$,
   (d) $\{(y, x) \mid (x \in X) \wedge (y \in Y)\}$,
   (e) none of the above.

9. Exercise 9. (4 points) Let $X = \{3, 4\}$ and $Y = \{1, 2\}$. The Cartesian product $X \times Y$ is
   (a) $X \times Y = \{1, 2, 3, 4\}$,
   (b) $X \times Y = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$,
   (c) $X \times Y = \{(3, 1), (3, 2), (4, 1), (4, 2)\}$,
   (d) $X \times Y = \{(1, 2), (3, 4)\}$,
   (e) none of the above.

10. Exercise 10. (4 points) Let $X = \{2, 4\}$. The power set of X is:
   (a) $P(X) = \big\{2, 4, \{2, 4\}\big\}$.
   (b) $P(X) = \big\{\emptyset, 2, 4, \{2, 4\}\big\}$.
   (c) $P(X) = \big\{\{2\}, \{4\}, \{2, 4\}\big\}$.
   (d) $P(X) = \big\{\emptyset, \{2\}, \{4\}, \{2, 4\}\big\}$.
   (e) none of the above.

11. Exercise 11. (12 points) Recall that $2^0 = 1$. Prove that for all $n \geq 1$,

$$\sum_{k=0}^{n-1} 2^k = 2^n - 1.$$

12. Exercise 12. (12 points) Consider the sequence $(a_n)_{n=1}^{\infty}$ recursively defined as $a_1 = 2$, $a_2 = 4$ and for all $n \geq 2$, $a_{n+1} = 4a_n - 4a_{n-1}$. For all $n \geq 1$, find a closed formula for a_n and prove that your conjecture is true.

13. Exercise 13. (8 points) Let $X = \{n \in \mathbb{Z} \mid (\exists k \in \mathbb{Z})[n = 6k]\}$ and $Y = \{n \in \mathbb{Z} \mid n \text{ is even}\}$. Prove that $X \subseteq Y$.

14. Exercise 14. (16 points)

(a) (8 points) Let X and Y be sets. Show that $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$.

(b) (8 points) Let $(X_i)_{i \in I}$ be a collection of sets. Show that $\overline{\bigcap_{i \in I} X_i} = \bigcup_{i \in I} \overline{X}_i$.

15. Exercise 15. (12 points) For all $n \geq 1$, let $A_n = \left[0, 2 - \frac{1}{n}\right]$.

(a) (2 points) $\bigcup_{n=1}^{\infty} A_n =$.

(b) (4 points) Prove that your answer to part (a) is correct.
(Hint: Remember that for all $y > 0$ there exists an integer $k \geq 1$ such that $0 < \frac{1}{k} \leq y$.)

(c) (2 points) $\bigcap_{n=1}^{\infty} A_n =$ ________________ .

(d) (4 points) Prove that your answer to part (c) is correct.