TEXAS A&M UNIVERSITY

Course Syllabus

Course Information

Number:	MATH 689-601
Title:	Metric Invariants: Algorithmic and Geometric Applications
Time:	TR: 11:30 am-12:45 pm
Location:	ONLINE via ZOOM
Credit hours:	3
Webpage:	`http://www.math.tamu.edu/~florent/teaching/689Fall20.html`

Instructor Details

Instructor:	Florent BAUDIER
Office:	Blocker 623B
Phone:	+1-979-845-3261 (Math Department Teaching Operations)
E-mail:	`florent@math.tamu.edu`
Office hours:	by appointment

Course Description

Name:	Special Topics. Metric Invariants: Algorithmic and Geometric Applications.
Description:	We will study sophisticated and powerful metric invariants and their connections with modern cutting-edge techniques in the design of algorithms, in geometric group theory, and in Alexandrov geometry. In particular we will showcase interesting applications of harmonic analysis, martingales techniques, random walks on graphs and Markov chains in general, to the theory of metric invariants and their applications to geometry and theoretical computer science.

Prerequisities

The course will be designed to be as inclusive as possible to most of the science and engineering majors. Students majoring CSCE, ECEN, ISEN, STAT, are strongly encourage to enroll and to express their interest directly to the instructor. For instance NO programming background is required and NO particular mathematics class is a prerequisite. However, it is expected that students wishing to enroll in this class are familiar with:

- basics of linear algebra (as can be found in MATH 304, 309, 311, 323)
- basics of discrete probability theory (e.g. STAT 211, ECEN 303, or MATH 411)
- proof writing (e.g. MATH 300 or equivalent) and discrete mathematics (as can be found in MATH 302 or CSCE 222/ECEN 222).

Course Learning Outcomes

Our ability to faithfully represent an a priori unstructured geometric object into a highly structured one has proven to be one of the most powerful lock breaker to some of the hardest problems, whether in pure mathematics or theoretical computer science. One of the main goal of this course is to give a thorough exposure to numerous techniques and ideas that rely on this geometric approach and that are fundamental in data science and in pure mathematics. We expect the course to be of interest to students from various majors and scientific backgrounds. One objective of this course will be to foster interactions between the students from various background. The very active and popular field of research to be presented in this course is rapidly evolving. Numerous important open problems will be discussed with the ultimate goal to trigger the interest of starting graduate students (and senior undergraduate students who qualify) into pursuing research work in this direction.

Textbook and/or Resource Materials

The instructor will provide lecture notes. References to textbooks, lecture notes, research papers will be posted on the course webpage when relevant.

Grading Policy

Due to the COVID-19 pandemic, the grading policy is subject to be modified (only in students' favor) in order to accommodate the very fluid situation and student-specific situations.

Homework:	There will be at most 5 short homework assignments (about every 3 weeks).
Q-A meeting:	Close to the end of the semester I will schedule a 30'-meeting with each student enrolled in the class. During this meeting you will go over a topic discussed in class and I will assess your understanding of the class material during an informal discussion.
Attendance	Attendance will be collected via ZOOM and you will accrue 1 point per lecture attended, up to a maximum of 20 points.
Final Grade:	The final grade will be computed as follows: final grade = homework 50%+ Q-A 30%+ attendance 20%.
Late work policy:	If your homework is submitted x days late it will cost a penalty of x points.
Grading:	A 90-100% B 80-89% C 70-79% D 60-69% F 0-59%
Appeal:	Due to FERPA privacy issues, I cannot discuss grades over email or phone. If you have a question about your grade, please schedule a meeting with me.

Course Schedule

Week 1: Sparsest Cut Problem (metric cones, LP relaxation, SDP relaxation, distortion, network flows)

Week 2: Sparsest Cut Problem (metric cones, LP relaxation, SDP relaxation, distortion, network flows)

Week 3: L_1-distortion (trees, stochastic embeddings into trees, series-parallel graphs, Earthmover distance, buy-at-bulk networks...)

Week 4: L_1-distortion (trees, stochastic embeddings into trees, series-parallel graphs, Earthmover distance, buy-at-bulk networks...)

Week 5: Distortion lower bounds via Fourier analysis

Week 6 Euclidean distortion (Hamming cubes, diamond graphs, binary trees), SDP program

Week 7 Nearest Neighbor Search (hashing, dimension reduction, ANN in Euclidean space and Hamming cubes)

Week 8 Nearest Neighbor Search (hashing, dimension reduction, ANN in Euclidean space and Hamming cubes)

Week 9 Ribe program (Ribe's rigidity theorem, Enflo problem)

Week 10 Ribe program (Bourgain's metric characterization of super-reflexivity, intrinsic vs extrinsic metric invariants)

Week 11: Gromov's coarse universality problem (Mendel-Naor metric cotype, barycentric spaces, Hadamard spaces, Pisier's martingale inequality...)

Week 12: Gromov's coarse universality problem (Mendel-Naor metric cotype, barycentric spaces, Hadamard spaces, Pisier's martingale inequality...)

Week 13: Markov convexity (connection with uniform convexity, Markov convexity of trees, Laakso graphs, Heisenberg group)

Week 14: Markov convexity (connection with uniform convexity, Markov convexity of trees, Laakso graphs, Heisenberg group)

Week 15: Thanksgiving Holiday

Course Content

Basics of metric geometry and embedding theory

- metric and normed spaces, Hilbert and Lebesgue spaces, graphs and groups as geometric objects,
- bi-Lipschitz embeddings and distortion
- coarse embeddings and compression
- embedding techniques
- Johnson-Lindenstrauss dimension reduction technique

Algorithmic problems

- Sparsest Cut
- Network flows
- Buy-at-bulk networks
- Nearest Neighbor Search
- Computing the Euclidean distortion
- Flows and cuts sparsifiers (if time permits)
- Compressive Sensing (if time permits)

Ribe program and metric invariants

- Ribe's rigidity theorem
- Rademacher type and Rademacher cotype
- uniform convexity and uniform smoothness
- Extrinsic and intrinsic metric invariants

Metric type

- Enflo type and the geometry of the Hamming cubes
- Pisier's inequality and Enflo's problem
- Solution to Enflo's problem (this 40-year old problem was solved in Spring 2020)
- Bourgain-Milman-Wolfson type (if time permits)

Metric cotype and applications

- Mendel-Naor metric cotype and the geometry of ℓ_∞-grids
- metric cotype of barycentric metric spaces
- nonlinear martingales and Pisier's martingale cotype inequality in barycentric metric spaces
- Eskenazis-Mendel-Naor solution of Gromov's problem about coarse universality of Alexandrov spaces of nonpositive curvature (problem solved in 2018)

Markov convexity and applications

- Markov convexity and the geometry of trees, diamond graphs, Laakso graphs, Heisenberg group.
- Markov convexity and uniform convexity
- polynomial time approximation algorithm to compute the ℓ_p-distortion of tree metrics (if time permits)
- tree dichotomy (it time permits)

University Policies

Attendance Policy:

- The university views class attendance and participation as an individual student responsibility. Students are expected to attend class and to complete all assignments.
- Please refer to Student Rule 7 in its entirety for information about excused absences, including definitions, and related documentation and timelines.
- Providing a fake or falsified doctor's note or other falsified documentation is considered academic dishonesty, will be reported to the Aggie Honor Council, and will result in an F in the course.

Makeup Work Policy:

- Students will be excused from attending class on the day of a graded activity or when attendance contributes to a student's grade, for the reasons stated in Student Rule 7, or other reason deemed appropriate by the instructor.
- Please refer to Student Rule 7 in its entirety for information about makeup work, including definitions, and related documentation and timelines.
- Absences related to Title IX of the Education Amendments of 1972 may necessitate a period of more than 30 days for make-up work, and the timeframe for make-up work should be agreed upon by the student and instructor" (Student Rule 7, Section 7.4.1).
- "The instructor is under no obligation to provide an opportunity for the student to make up work missed because of an unexcused absence" (Student Rule 7, Section 7.4.2).
- Students who request an excused absence are expected to uphold the Aggie Honor Code and Student Conduct Code. (See Student Rule 24.)

Academy Integrity Statement and Policy:

- "An Aggie does not lie, cheat or steal, or tolerate those who do."
- "Texas A&M University students are responsible for authenticating all work submitted to an instructor. If asked, students must be able to produce proof that the item submitted is indeed the work of that student. Students must keep appropriate records at all times. The inability to authenticate one's work, should the instructor request it, may be sufficient grounds to initiate an academic misconduct case" (Section 20.1.2.3, Student Rule 20).
- You can learn more about the Aggie Honor System Office Rules and Procedures, academic integrity, and your rights and responsibilities at aggiehonor.tamu.edu.

Americans with Disability Act (ADA) Policy:

- Texas A&M University is committed to providing equitable access to learning opportunities for all students. If you experience barriers to your education due to a disability or think you may have a disability, please contact Disability Resources in the Student Services Building or at (979) 845-1637 or visit disability.tamu.edu. Disabilities may include, but are not limited to attentional, learning, mental health, sensory, physical, or chronic health conditions. All students are encouraged to discuss their disability related needs with Disability Resources and their instructors as soon as possible.

Title IX and Statement on Limits to Confidentiality

- Texas A&M University is committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws prohibit gender-based discrimination and sexual harassment, including sexual assault, sexual exploitation, domestic violence, dating violence, and stalking.
- With the exception of some medical and mental health providers, all university employees (including full and part-time faculty, staff, paid graduate assistants, student workers, etc.) are Mandatory Reporters and must report to the Title IX Office if the employee experiences, observes, or becomes aware of an incident that meets the following conditions (see University Rule 08.01.01.M1):
  - The incident is reasonably believed to be discrimination or harassment. The incident is alleged to have been committed by or against a person who, at the time of the incident, was (1) a student enrolled at the University or (2) an employee of the University.
  - Mandatory Reporters must file a report regardless of how the information comes to their attention – including but not limited to face-to-face conversations, a written class assignment or paper, class discussion, email, text, or social media post. Although Mandatory Reporters must file a report, in most instances, you will be able to control how the report is handled, including whether or not to pursue a formal investigation. The University's goal is to make sure you are aware of the range of options available to you and to ensure access to the resources you need.
- Students wishing to discuss concerns in a confidential setting are encouraged to make an appointment with Counseling and Psychological Services (CAPS).
- Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the University's Title IX webpage.

Statement on Mental Wealth and Wellness

- Texas A&M University recognizes that mental health and wellness are critical factors that influence a student's academic success and overall wellbeing. Students are encouraged to engage in proper self-care by utilizing the resources and services available from Counseling & Psychological Services (CAPS). Students who need someone to talk to can call the TAMU Helpline (979-845-2700) from 4:00 p.m. to 8:00 a.m. weekdays and 24 hours on weekends. 24-hour emergency help is also available through the National Suicide Prevention Hotline (800-273-8255) or at suicidepreventionlifeline.org.

COVID-19 Temporary Amendment to Minimum Syllabus Requirements

Campus Safety Measures

To promote public safety and protect students, faculty, and staff during the coronavirus pandemic, Texas A&M University has adopted policies and practices for the Fall 2020 academic term to limit virus transmission. Students must observe the following practices while participating in face-to-face courses and course-related activities (office hours, help sessions, transitioning to and between classes, study spaces, academic services, etc.):

- Self-monitoring–Students should follow CDC recommendations for self-monitoring. Students who have a fever or exhibit symptoms of COVID-19 should participate in class remotely and should not participate in face-to-face instruction.
- Face Coverings–Face coverings (cloth face covering, surgical mask, etc.) must be properly worn in all non-private spaces including classrooms, teaching laboratories, common spaces such as lobbies and hallways, public study spaces, libraries, academic resource and support offices, and outdoor spaces where 6 feet of physical distancing is difficult to reliably maintain. Description of face coverings and additional guidance are provided in the Face Covering policy and Frequently Asked Questions (FAQ) available on the Provost website.
- Physical Distancing–Physical distancing must be maintained between students, instructors, and others in course and course-related activities.
- Classroom Ingress/Egress–Students must follow marked pathways for entering and exiting classrooms and other teaching spaces. Leave classrooms promptly after course activities have concluded. Do not congregate in hallways and maintain 6-foot physical distancing when waiting to enter classrooms and other instructional spaces.
- To attend a face-to-face class, students must wear a face covering (or a face shield if they have an exemption letter). If a student refuses to wear a face covering, the instructor should ask the student to leave and join the class remotely. If the student does not leave the class, the faculty member should report that student to the Student Conduct office for sanctions. Additionally, the faculty member may choose to teach that day's class remotely for all students.

Personal Illness and Quarantine

- Students required to quarantine must participate in courses and course-related activities remotely and must not attend face-to-face course activities. Students should notify their instructors of the quarantine requirement. Students under quarantine are expected to participate in courses and complete graded work unless they have symptoms that are too severe to participate in course activities.
- Students experiencing personal injury or Illness that is too severe for the student to attend class qualify for an excused absence (See Student Rule 7, Section 7.2.2.) To receive an excused absence, students must comply with the documentation and notification guidelines outlined in Student Rule 7. While Student Rule 7, Section 7.3.2.1, indicates a medical confirmation note from the student's medical provider is preferred, for Fall 2020 only, students may use the Explanatory Statement for Absence from Class form in lieu of a medical confirmation. Students must submit the Explanatory Statement for Absence from Class within two business days after the last date of absence.

Operational Details fo Fall 2020 Courses

- For additional information, please review the FAQ on Fall 2020 courses at Texas A&M University.

Miscellaneous

Copyright Policy

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