Test II

Instructions: Show all work in your bluebook. Calculators that do not do linear algebra are allowed.

1. (20 pts.) For the matrix $A$ below, find a basis for the row space, a basis for the column space, and a basis for the null space. Also, determine the rank and nullity of $A$.

$$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{pmatrix}$$

2. (15 pts.) Find the standard matrix representation for the linear transformation $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that rotates a vector $90^\circ$ in the clockwise direction and then reflects it about the $x_2$-axis.

3. (15 pts.) Consider the bases for $P_2$ defined by

$$F = [x + 1, x - 1] \quad \text{and} \quad G = [1, 2 - x].$$

Find the transition matrix $S_{F \rightarrow G}$ that takes coordinates relative to $F$ into ones relative to $G$.

4. (15 pts.) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be linear. If $T((1, 1)^T) = (-1, 2)^T$ and $T((2, -1)^T) = (3, 2)^T$, find $T((4, 1)^T)$.

5. Let $L : P_3 \rightarrow P_3$ be defined by $L(p) = (x + 1)p' - 2p$.

(a) (5 pts.) Show that $L$ is linear.

(b) (10 pts.) Find the kernel and range of $L$.

(c) (10 pts.) Find the matrix $A$ that represents $L$ relative to $E$-coordinates, where $E = [x^2, x, 1]$.

(d) (5 pts.) Find the matrix $B$ that represents $L$ relative to $H$-coordinates, where $H = [1, 1 + x, 1 - x - x^2]$. (You may use your answer to the previous question, You do not need to invert or multiply the matrices involved.)

6. (5 pts.) Let $A$ be an $n \times (n + 1)$ matrix. If the nullity of $A$ is 2, do the columns of $A$ form a basis for $\mathbb{R}^n$? Why?