Instructions: Show all work in your bluebook. Calculators that do linear algebra or calculus are not allowed.

1. Define the following:
   (a) (5 pts.) $C[a,b]$, and its operations of addition and scalar multiplication.
   (b) (5 pts.) $\text{Span}\{v_1, \ldots, v_n\}$.

2. (10 pts.) Find the adjacency matrix $A$ for the graph below, and compute the first row of $A^2$. What do these entries tell you about walks of length 2 that start from $V_1$?

3. (20 pts.) A linear system $Ax = b$ has the augmented matrix $[A|b]$ given below. Use row reduction to solve the system. Also, identify the leading variables and free variables, and find $N(A)$.

4. (10 pts.) Let $S = \{(x_1, x_2, x_3)^T \in \mathbb{R}^3 | x_1 - 2x_2 = x_3 \} \subset \mathbb{R}^3$. Determine whether or not $S$ is a subspace of $\mathbb{R}^3$.

5. Let $C = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 1 \\ -2 & -5 & 3 \end{pmatrix}$.
   (a) (15 pts.) Find $C^{-1}$ by row reducing the augmented matrix $[C|I]$, keeping careful track of the row operations that you use.
   (b) (10 pts.) By inspecting these row operations, give elementary matrices $E, E', E''$ such that $E''E'EC = U$, where $U$ is upper triangular.
   (c) (10 pts.) Find $\det C$, using any method.
6. (15 pts.) Do one of the following problems.

(a) Define the term inverse of an $n \times n$ matrix $A$. Show that if $A$ and $B$ are invertible, then $AB$ is, too, and $(AB)^{-1} = B^{-1}A^{-1}$.

(b) Let $A$ be an $n \times n$ matrix. Show that if $Ax = 0$ has only $x = 0$ as a solution, then $A$ is row equivalent to the identity.

(c) Let $A$ be an $n \times n$ matrix. Show that if $A$ is row equivalent to the identity, then $A$ is nonsingular.