Test II

Instructions: *Show all work in your bluebook. You may use a calculator for numerical computations. You may* not *use a graphing calcular or a calculator that can do symbolics.*

1. **(20 pts.)** Let $h(t) = \begin{cases} e^{-2t} & t \geq 0, \\ 0 & t < 0 \end{cases}$ and $f(t) = \begin{cases} 1 & 0 \leq t \leq \pi, \\ 0 & t < 0 \text{ or } t > \pi \end{cases}$. If H is the filter $H[f] = h * f$, find $H[f]$. Is H causal? Find the *system function*, $\sqrt{2\pi}\hat{h}$.

2. Let $\mathcal{F}_n$ be the DFT for n-periodic sequences (signals).

   (a) **(10 pts.)** Briefly describe the FFT. Why is it *fast*?

   (b) **(10 pts.)** Find $\mathcal{F}_4[a]$ if $a = (-2, 1, 0, 1)$. Note: for $n = 4$, $\overline{w} = -i$.

   (c) **(10 pts.)** State the (circular) convolution theorem for the DFT, and use it and part **2b** above to find the eigenvalues of the circulant matrix A whose first column is $a^T = [-2\ 1\ 0\ 1]^T$.

3. **(20 pts.)** Define *multiresolution analysis* (MRA). In the case of the Haar MRA, state or define the following items: the approximation spaces, V_j, the scaling function ϕ, the two-scale (scaling) relation, and the wavelet ψ. State what H, L are in Fig. 1.

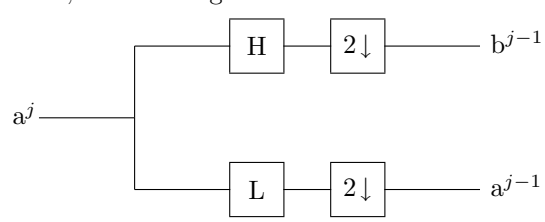

Figure 1: Haar decomposition diagram.

4. **(15 pts.)** Reconstruct $f \in V_3$, given these coefficients in its Haar wavelet decomposition:

$$a^1 = [3/2, -1] \quad b^1 = [-1, -3/2] \quad b^2 = [-3/2, -3/2, -1/2, -1/2].$$

   The first entry in each list corresponds to $k = 0$. Sketch f.

5. **(15 pts.)** Prove the Sampling Theorem: *Let f be band-limited. That is, suppose that $\hat{f}(\lambda)$ is piecewise smooth, continuous, and that $\hat{f}(\lambda) = 0$ for $|\lambda| > \Omega$, where Ω is some fixed, positive frequency. Then f has the following series expansion:*

$$f(t) = \sum_{j=-\infty}^{\infty} f(j\pi/\Omega) \frac{\sin(\Omega t - j\pi)}{\Omega t - j\pi}. \tag{1}$$