Test 2

Instructions: *Show all work in your bluebook. Cell phones, laptops, calculators that do linear algebra or calculus, and other such devices are* not *allowed..*

1. Statements of definitions and theorems.
   (a) **(5 pts.)** The discrete signal space ℓ^2 and the Z-transform on ℓ^2.
   (b) **(5 pts.)** Linear, time-invariant filter.
   (c) **(5 pts.)** Band-limited function and Nyquist *rate*.
   (d) **(5 pts.)** The number of multiplications required for the finding the FFT of a signal of length N, where N is a power of 2.

2. **(15 pts.)** Find $\mathcal{F}[e^{-|t|}]$.

3. **(15 pts.)** Find $\mathcal{F}^{-1}[\hat{f}]$, where $\hat{f}(\lambda) := \begin{cases} 1 & 0 \le \lambda \le \pi\,, \\ 0 & \lambda < 0 \text{ or } \lambda > \pi\,. \end{cases}$.

4. **(20 pts.)** Let $h(t) = \begin{cases} \alpha e^{-\alpha t} & t \ge 0\,, \\ 0 & t < 0 \end{cases}$ be the impulse response (IR) for the Butterworth filter $L[f] = h * f$. Find $L[f]$, where

$$f(t) = \begin{cases} e^{-t}\sin(3t) & 0 \le t \le 4\pi\,, \\ 0 & t < 0 \text{ or } t > 4\pi\,. \end{cases}$$

5. **(15 pts.)** Let $\mathcal{S}_n$ be the space of n-periodic sequences. If $y \in \mathcal{S}_n$ and if $z \in \mathcal{S}_n$ is defined by $z_j = y_{j+1}$, show that $\hat{z}_k = w^k \hat{y}_k$, where $w = e^{2\pi i/n}$.

6. **(15 pts.)** Do *one* of the following:
   (a) State the Sampling Theorem and sketch a proof of it.
   (b) State and prove the Convolution Theorem for the discrete Fourier transform.
   (c) State and sketch a proof of the Uncertainty Principle. (You may do the case in which $a = 0$ and $\alpha = 0$.)

Integrals

1. $\int udv = uv - \int vdu$
2. $\int \frac{dt}{t} = \ln\left|t\right| + C$
3. $\int e^{at}dt = \frac{1}{a}e^{at} + C$
4. $\int t^n e^{at}dt = \frac{1}{a}t^n e^{at} - \frac{n}{a}\int t^{n-1}e^{at}dt$
5. $\int e^{at}\cos(bt)dt = \frac{e^{at}}{a^2+b^2}\left(a\cos(bt) + b\sin(bt)\right) + C$
6. $\int e^{at}\sin(bt)dt = \frac{e^{at}}{a^2+b^2}\left(a\sin(bt) - b\cos(bt)\right) + C$
7. $\int t\sin(t)dt = \sin(t) - t\cos(t) + C$
8. $\int t\cos(t)dt = \cos(t) + t\sin(t) + C$
9. $\int \tan(at)dt = \dfrac{1}{a}\ln\left|\sec(at)\right| + C$
10. $\int \cot(at)dt = \dfrac{1}{a}\ln\left|\sin(at)\right| + C$
11. $\int \sec(at)dt = \dfrac{1}{a}\ln\left|\sec(at) + \tan(at)\right| + C$
12. $\int \csc(at)dt = \dfrac{1}{a}\ln\left|\csc(at) - \cot(at)\right| + C$
13. $\int \dfrac{dt}{t^2 + a^2} = \dfrac{1}{a}\arctan(t/a) + C$