Test 2

Instructions: Show all work in your bluebook. Cell phones, laptops, calculators that do linear algebra or calculus, and other such devices are not allowed.

1. Define the terms below.
   
   (a) (10 pts.) Define the term multiresolution analysis. For the Shannon MRA, define the $V_j$’s and $\phi$.
   
   (b) (5 pts.) Define the term time-invariant filter.
   
   (c) (5 pts.) Define the fast Fourier transform and briefly state how it is related to the discrete Fourier transform.

2. (20 pts.) Let $h(t) = \begin{cases} 1/5 & 0 \leq t \leq 5, \\ 0 & t < 0 \text{ or } t > 5 \end{cases}$ be the impulse response for $L[f] = h * f$. Find $L[f]$, where $f = \begin{cases} e^{-3t} & t \geq 0, \\ 0 & t < 0. \end{cases}$ Find the system (frequency) response function, $\sqrt{2\pi} \hat{h}$. Is the filter causal? Why or why not? (One or two sentences will suffice.)

3. (20 pts.) Let $y = (\cdots y_0, y_1, \cdots, y_{n-1} \cdots) \in S_n$, where $S_n$ is the space of $n$-periodic sequences. Show that if the $y_j$’s are real, then $\hat{y}_{n-k} = \overline{\hat{y}_k}$

4. (20 pts.) The level $j = 2$ Haar MRA approximation coefficients for $f_2 \in V_2$ are $a^2 = [3, 1, -2, 0, -3, 9, -3]$, $k = 0 \ldots 6$; for $k < 0$ or $k > 6$, $a_k^2 = 0$. Decompose $f_2$ down to level $j = 0$.

5. (20 pts.) Do one of the following.
   
   (a) For the Haar MRA, define $V_j$ and prove this: $f(x)$ belongs to $V_j$ if and only if $f(2^{-j}x)$ belongs to $V_0$.
   
   (b) Suppose that $x$ is an $n$-periodic sequence – i.e., $x \in S_n$. Show that $\sum_{j=m}^{n-1} x_j = \sum_{j=0}^{n-1} x_j$.
   
   (c) State and prove the Whittaker-Shannon Sampling Theorem.