Test 3

Instructions: *Show all work in your bluebook. Cell phones, laptops, calculators that do linear algebra or calculus, and other such devices are* not *allowed.*

1. **(20 pts.)** Define the term *multiresolution analysis* (MRA). For the Shannon MRA, V_j is the subspace of $f \in L^2$ such that $\mathrm{supp}(\hat{\mathrm{f}})$ is contained in the interval in $[-2^j\pi, 2^j\pi]$, and $\phi(x) = \mathrm{sinc}(x) := \frac{\sin(\pi x)}{\pi x} \in V_0$ is the scaling function. Find the p_k's in the scaling relation.

2. **(20 pts.)** For any MRA, where the p_k's are real, the decomposition formulas are $a_\ell^{j-1} = \frac{1}{2}\sum_{k\in\mathbb{Z}} p_{k-2\ell}\, a_k^j$ and $b_\ell^{j-1} = \frac{1}{2}\sum_{k\in\mathbb{Z}}(-1)^k p_{1-k+2\ell}\, a_k^j$. Derive the filter form shown in Figure 1 – i.e., start with the formulas and work your way to $a^{j-1} = DLa^j$ and $b^{j-1} = DHa^j$. Be sure to define H, L, and $2\!\downarrow$ $(=D)$.

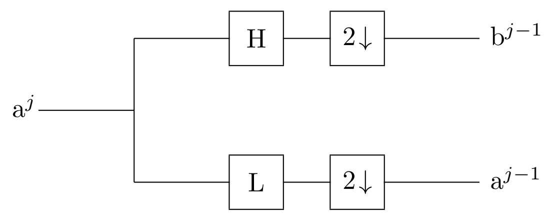

Figure 1: Wavelet decomposition diagram.

3. **(20 pts.)** Reconstruct $f \in V_3$, given these coefficients in its Haar wavelet decomposition: $a^2 = [1/2, 2, 5/2, -3/2]$, $b^2 = [-3/2, -1, 1/2, -1/2]$. The first entry in each list corresponds to $k = 0$.

4. **(20 pts.)** The scaling relation for an MRA is $\phi(x) = \sum_{k=-\infty}^{\infty} p_k \phi(2x-k)$. Show that $\hat{\phi}(\xi) = P(e^{-i\xi/2})\hat{\phi}(\xi/2)$, where $P(z) = \frac{1}{2}\sum_{k=-\infty}^{\infty} p_k z^k$. State the conditions $P(z)$ should satisfy for this construction to yield a scaling function. Show that these conditions imply $P(-1) = 0$.

5. **(20 pts.)** Do <u>*one*</u> of the following.

   (a) The Daubechies scaling relation has the form $\phi(x) = p_0\phi(2x) + p_1\phi(2x-1) + p_2\phi(2x-2) + p_3\phi(2x-3)$. Show that the support of $\phi(x)$ is $[0, 3]$.

   (b) Show that the Daubechies $N = 2$ wavelet decomposition "reproduces" linear polynomials; that is, if $f(x) = Ax + B$, then $b_k^j = 0$ for all j, k.