List of Errata for the Book
A Mathematical Introduction to Compressive Sensing

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This list was last updated on September 27, 2021. If you see further errors, please send us an e-mail at `foucart@tamu.edu` and `rauhut@mathc.rwth-aachen.de`.

Chapter 2

- Page 45, Remark 2.8 is incorrect, hence Exercise 2.2 should be discarded. Indeed, the result of Proposition 2.7 holds with the sharper constant $k^{1/p}$. Here is the justification:

  Proof. Let $t > 0$. For each $i \in [k]$, we consider $t_i := \|\mathbf{x}^i\|_{p,\infty}/c$ for some c chosen so that $t_1 + \cdots + t_k = t$, i.e., $c = (\|\mathbf{x}^1\|_{p,\infty} + \cdots + \|\mathbf{x}^k\|_{p,\infty})/t$. If $|x_j^1 + \cdots + x_j^k| \geq t$ for some $j \in [N]$, then we have $|x_j^i| \geq t_i$ for some $i \in [k]$. This means that

  $$\{j \in [N] : |x_j^1 + \cdots + x_j^k| \geq t\} \subset \bigcup_{i \in [k]} \{j \in [N] : |x_j^i| \geq t_i\} .$$

  We derive

  $$\mathrm{card}\{j \in [N] : |x_j^1 + \cdots + x_j^k| \geq t\} \leq \sum_{i \in [k]} \frac{\|\mathbf{x}^i\|_{p,\infty}^p}{t_i^p} = kc^p.$$

  According to the definition of the weak ℓ_p-quasinorm of $\mathbf{x}^1 + \cdots + \mathbf{x}^k$, we obtain

  $$\|\mathbf{x}^1 + \cdots + \mathbf{x}^k\|_{p,\infty} \leq k^{1/p} c = k^{1/p} \big(\|\mathbf{x}^1\|_{p,\infty} + \cdots + \|\mathbf{x}^k\|_{p,\infty}\big).$$

  This is the required result. □

- Page 48, Lines 26 and 31: replace 's-sparse $\mathbf{x} \in \mathbb{C}^N$' by '$\mathbf{x} \in \mathbb{C}^N$ with $\|\mathbf{x}\|_0 = s$', otherwise the implication $(b) \Rightarrow (a)$ may not be true

- Page 51, Theorem 2.15: the statement concerns s-sparse vectors, not $2s$-sparse vectors

- Page 51, Line 22: '$\hat{p} * \hat{x} = \widehat{p \cdot x} = 0$' should read '$\hat{p} * \hat{x} = N\, \widehat{p \cdot x} = 0$'

- Page 52, Line 11: 'so that the trigonometric polynomial q vanishes on S': this statement is only valid if the support of x is exactly S; to repair the argument, take $\hat{q}(1), \ldots, \hat{q}(s)$ as a solution of the linear system with a maximum number of consecutive zero values for $\hat{q}(s)$, $\hat{q}(s-1)$, ... (this is done by solving a sequence of linear systems), then replace s by $\|x\|_0$ and S by $\mathrm{supp}(x)$ in Lines 9-12

Chapter 3

- Page 66, Lines 28-29: a vector $\mathbf{u}$ appears twice — it should be replaced by $\mathbf{v}$
- Page 74, Exercise 3.4: the condition about the invertibility of the submatrices is not necessary
- Page 74, Exercise 3.8: one may assume that the matrix $\mathbf{A} \in \mathbb{C}^{m \times N}$ is of full rank $m < N$
- Page 74, Exercise 3.9: 'cannot be recovered via the orthogonal matching pursuit algorithm' should really read 'cannot be recovered in one iteration of the orthogonal matching pursuit algorithm'
- Page 75, Exercise 3.10: a complex conjugation is missing on line 8, which should read

$$\Delta_n = \|\mathbf{A}(\mathbf{x}^{n+1} - \mathbf{x}^n)\|_2^2 = \overline{x_{j^{n+1}}^{n+1}}(\mathbf{A}^*(\mathbf{y} - \mathbf{A}\mathbf{x}^n))_{j^{n+1}}$$

Chapter 4

- Page 109, Exercise 4.20(b): one should read '$\mathbf{M} \in \mathbb{C}^{n_1 \times n_2}$' instead of '$\mathbf{M} \in \ker \mathcal{A} \setminus \{\mathbf{0}\}$'; the occurrences'$\|\mathbf{e}\|_2$' and '$\|\mathcal{A}(\mathbf{Z}) - \mathbf{y}\|_2$' of an ℓ_2-norm should be replaced by '$\|\mathbf{e}\|$' and '$\|\mathcal{A}(\mathbf{Z}) - \mathbf{y}\|$' with a general norm; and 'quadratically constrained' should be rephrased as 'inequality-constrained'

Chapter 5

- Page 113, Definition 5.5: read '$0 \leq c < 1$' instead of just '$c \geq 0$' (to exclude the case of a repeated vector)
- Page 120, Theorem 5.12: 'For $m \geq 3$' should read 'For $m > 3$' (indeed, when $m = 3$, equiangular systems of $N = m(m+1)/2$ vectors in $\mathbb{R}^m$ exist — see Exercise 5.5 — yet $m+2$ is not the square of an odd integer); in the proof of the theorem, one should also verify that Σ_1 and Σ_2 are nonzero, but if they were, then $\Sigma_1 - \sqrt{m+1}\,\Sigma_2 = 0$ would mean that $\sqrt{m+2} = 1/c$ is the other eigenvalue of $\mathbf{B}$, namely $(N/m - 1)/c = ((m+1)/2 - 1)/c$, which is impossible when $m > 3$

Chapter 6

- Page 134, Line 7: 'the interval $[1 - \delta_s, 1 + \delta_s]$' should read 'the interval $[\sqrt{1 - \delta_s}, \sqrt{1 + \delta_s}]$'
- Page 134, Line 14: 'relative $\ell_2(\mathbb{R})$' should read 'relative to $\ell_2(\mathbb{R})$'

- Pages 139-140, Proof of Theorem 6.8: more care is required to deal with the fact that the last block $\mathbf{A}_n$ may have less than t columns — one should establish $\mathrm{tr}(\mathbf{H}) \geq N(1-\delta_s)$ instead of (6.10) and $\mathrm{tr}(\mathbf{H})^2 \leq mN\left((n-1)\delta_s^2 + (1+\delta_s)^2\right)$ instead of (6.11), while the rest of the argument remains unchanged

- Page 142, Line 15: replace 'Corollary 4.5' by 'Theorem 4.5'

- Page 146: in the last line of (6.23), $\|\mathbf{v}_{S_0}\|$ should read $\|\mathbf{v}_{S_0}\|_2$

- Page 158-159: In Proposition 6.24, Theorem 6.25, and Lemma 6.26, the assumption that $\mathbf{A}$ has ℓ_2-normalized columns should be added, as the argument is based on Lemma 3.3, which requires this assumption

- Page 161, Lines 15 and 16: δ_{s+n} should be δ_{s+s^0+n} — this implies that δ_{s+K} found in Lines 19, 21, 24, as well as on Page 163, Lines 4 and 5, should be δ_{s+s^0+K}, but there is no repercussion on the final result because $\alpha/\gamma < 1$ still holds

- Page 162, Lines 14 and 16: instead of $1-\nu$, read $1-1/\nu$

- Page 171, Exercise 6.7: replace 'the unit ball in ℓ_p' by 'the unit ball in ℓ_p^N'

- Page 173, Exercise 6.19: establish the result under the condition $\delta_{3s} < 1/2$, not $\delta_{3s} < 1/3$

- Page 173, Exercise 6.21: assume that all vectors and matrices are real-valued rather than complex-valued throughout the exercise

Chapter 7

- Page 190, Line 11: the extra parenthesis after $(-B_\ell)$ should be removed, so that it reads

$$\exp(\theta X_\ell) = f(X_\ell) = f(t(-B_\ell) + (1-t)B_\ell) \leq \ldots$$

- Page 191, Line 15: it should read 'from Hoeffding's inequality (Theorem 7.20)'

- Page 199, Line 1: 'Bernstein's inequality' instead of 'Bernstein inequality'

- Page 199, Exercise 7.3: the exponent 2 on the right-hand side of the desired inequality has to be replaced by $p/(p-1)$, so that it reads

$$\mathbb{P}\left(\left|\sum_{\ell=1}^{M} a_\ell X_\ell\right| > t\|\mathbf{a}\|_2\right) \geq c_p \frac{(\sigma^2 - t^2)^{p/(p-1)}}{\mu^{2p/(p-1)}}, \quad 0 \leq t \leq \sigma.$$

- Page 199, Exercise 7.6: the inequality to be proved is in fact

$$\mathbb{E}\exp\left(\frac{tX^2}{2c}\right) \leq \frac{1}{\sqrt{1-2t}},$$

which is valid for any (not necessarily nonnegative) $t \leq 1/2$

Chapter 8

- Page 219, Line 10: replace 'positive semidefinite' by 'positive definite', so that it reads '... is concave on the set of positive definite matrices.'

Chapter 9

- Page 289, Line 1: an expectation $\mathbb{E}$ is missing; the left-hand side of the inequality should read $\mathbb{E}\min_{\mathbf{z}\in\mathcal{N}(\mathbf{x})}\|\mathbf{g}-\mathbf{z}\|_2^2$
- Page 306, Fig. 9.2: the caption should include 'Image courtesy of Jared Tanner' instead of 'Image Courtesy by Jared Tanner'
- Page 306, Exercise 9.2: the inequality inside the probability should be strict, otherwise (9.61) is wrong for $\mathbf{x}=\mathbf{0}$
- Page 307, Exercise 9.6: a renormalization is missing — it is indeed the matrix $\dfrac{\sqrt{\pi/2}}{m}\mathbf{A}$ that satisfies the stated modified restricted isometry property.

Chapter 10

- Page 312, Line 8: there is a deplorable break at the end of this line — '$\lim_{m\to\infty} d^m(K,X)=0$' should appear as one block
- Page 313, Line 25: '$=0$' is missing after $\lambda_{2;0}(\mathbf{v})$, so that one should read $\lambda_{2;\lambda_1(\mathbf{v})}=\lambda_{2;0}(\mathbf{v})=0$
- Page 314, Line 23: 'quasi-triangle' should be 'quasi-triangle inequality'
- Page 321, Line 11: '1-separating' should be '1-separated'

Chapter 12

- Page 432, Exercise 12.9: the lone L should be an M here

Chapter 13

- Page 439, Lemma 13.4: replace lines 2 and 3 by
  For each $i\in R(S)$, let $\ell(i)\in S$ denote a fixed left vertex connected to i. Then the set

$$E'(S) := \{\overline{ji}\in E(S) : j\neq \ell(i)\} = E(S)\setminus\{\overline{\ell(i)i}, i\in R(S)\}$$

- Page 442, Line 3: '$j = \text{card}(R(J))$' should read '$j = \text{card}(J)$'
- Page 453, Line 1: replace m by m' in 'given $\mathbf{y} \in \mathbb{C}^m$, ...'
- Page 453, Line 5: it should be emphasized that the stated condition may not be met if the bipartite graph fails to be a lossless expander, so the algorithm is not well defined in this case
- Page 454, Lines 2, 3, 4, 6: $B_{k,j}$, B_{k,j^*}, $B_{\ell+1,j^*}$ should instead be $B'_{k,j}$, B'_{k,j^*}, $B'_{\ell+1,j^*}$
- Page 454, Line 7: the two sums should start at $k = 1$ and not at $k = 0$

Appendix A

- Page 525, Line 16: there is one '$\|\mathbf{A}\mathbf{v}_1\|_2$' too many in this displayed equation

Appendix B

- Page 544, Theorem B.4: 'interiors' should read 'relative interiors'
- Page 545, Remark B.5: instead of K_1, K_2 intersecting in only one point, the second application of Theorem B.4 requires that K_1, K_2 intersect in only one point not in the relative interior of K_1, i.e., $K_1 \cap K_2 = \{\mathbf{x}_0\}$ with $\mathbf{x}_0 \notin \text{ri}(K_1)$
- Page 547, Definition B.13: '$(\mathbf{x}_j)_{j \geq 1} \subset \mathbb{R}$' should read '$(\mathbf{x}_j)_{j \geq 1} \subset \mathbb{R}^N$'

References

- Page 614, Reference 503: 'Wakinm' should be 'Wakin'

Back cover

- Line 12: 'build' should be 'built'